One Equation for a Smooth Transition Between Two Line Segments: Application to the Boore and Thompson (2015) finite-fault factor

David M. Boore

Boore and Thompson (2015) (BT15 hereafter) gave the following equation for the finite-fault factor (h, but sometimes written as f_ff in my SMSIM programs).

$$\log h(\mathbf{M}) = \begin{cases} a_1 + b_1(\mathbf{M} - M_{t1}) & \mathbf{M} \leq M_{t1} \\ c_0 + c_1(\mathbf{M} - M_{t1}) + c_2(\mathbf{M} - M_{t1})^2 + c_3(\mathbf{M} - M_{t1})^3 & M_{t1} < \mathbf{M} < M_{t2} \\ a_2 + b_2(\mathbf{M} - M_{t2}) & \mathbf{M} \geq M_{t2} \end{cases} \quad (1)$$

This is equation (3) in BT15, with the correction that $h(\mathbf{M})$ on the left side of BT15 equation (3) should have been $\log h(\mathbf{M})$. The coefficients are given in Table 1 of BT15. After the publication of BT15, I realized that only five coefficients are needed to specify the model: the coefficients of the equations for the line segments and the range of $\mathbf{M}$ over which the transition takes place between the two segments. A subroutine in the SMSIM file *rv_td_subs.for* reads these five coefficients and then computes the coefficients needed for the BT15 equation (3). In the SMSIM program params file, the input coefficients are those in these equations:

Line 1:

$$\log h = c_1 + c_2 \mathbf{M} \tag{2)}$$

Line 2:

$$\log h = c_3 + c_4 \mathbf{M} \tag{3}$$

and the width of the transition $\Delta \mathbf{M}$. For some reason that I no longer remember, I used the same coefficient names for different things in BT15 and in SMSIM. I will use the coefficients names in equations (2) and (3) here.

Peter Stafford devised a single equation to provide a smooth transition between two geometrical spreading lines (the equation gives the log of the geometrical spreading function, the function being R to a power for each line). The equation is given in Stafford et al. (202x), with a generalization discussed in *geometrical_spreading_with_smooth_transition_between_linear_segments.v4.pdf*, available from a link in http://daveboore.com/daves_notes.html. Although the single function was used to represent geometrical spreading, it can also be used for any function made up of two lines joined by a smooth transition. This then applies to the finite-fault factor discussed in the first paragraph. Five coefficients are still needed: the equations of the two lines (four coefficients) and a parameter h_{RAT} which is a measure of the ratio of the smooth $h(\mathbf{M})$ to the $h(\mathbf{M})$ of the lines at

the magnitude where the two lines intersect ($\mathbf{M}_T$). h_{RAT} should be less than unity if the slope of line 2 is less than that of line 1, as it is in BT15, and vice versa if the slope of line 2 is greater than that for line 1. The closer h_{RAT} is to unity, the sharper the transition. The equation for $h(\mathbf{M})$ is given by:

$$\log h(\mathbf{M}) = \log h_T + c_2(\mathbf{M} - \mathbf{M}_T) + (c_4 - c_2)\frac{1}{\xi}\log\left[\frac{10^{\xi(\mathbf{M} - \mathbf{M}_T)} + 1}{2}\right] \tag{4}$$

where

$$\mathbf{M}_T = \left(\frac{c_1 - c_3}{c_4 - c_2}\right) \tag{5}$$

is the magnitude at which the two lines intersect,

$$h_X = 10^{(c_1 + c_2 \mathbf{M}_T)}, \tag{6}$$

$$h_T = h_{RAT} h_X, \tag{7}$$

and

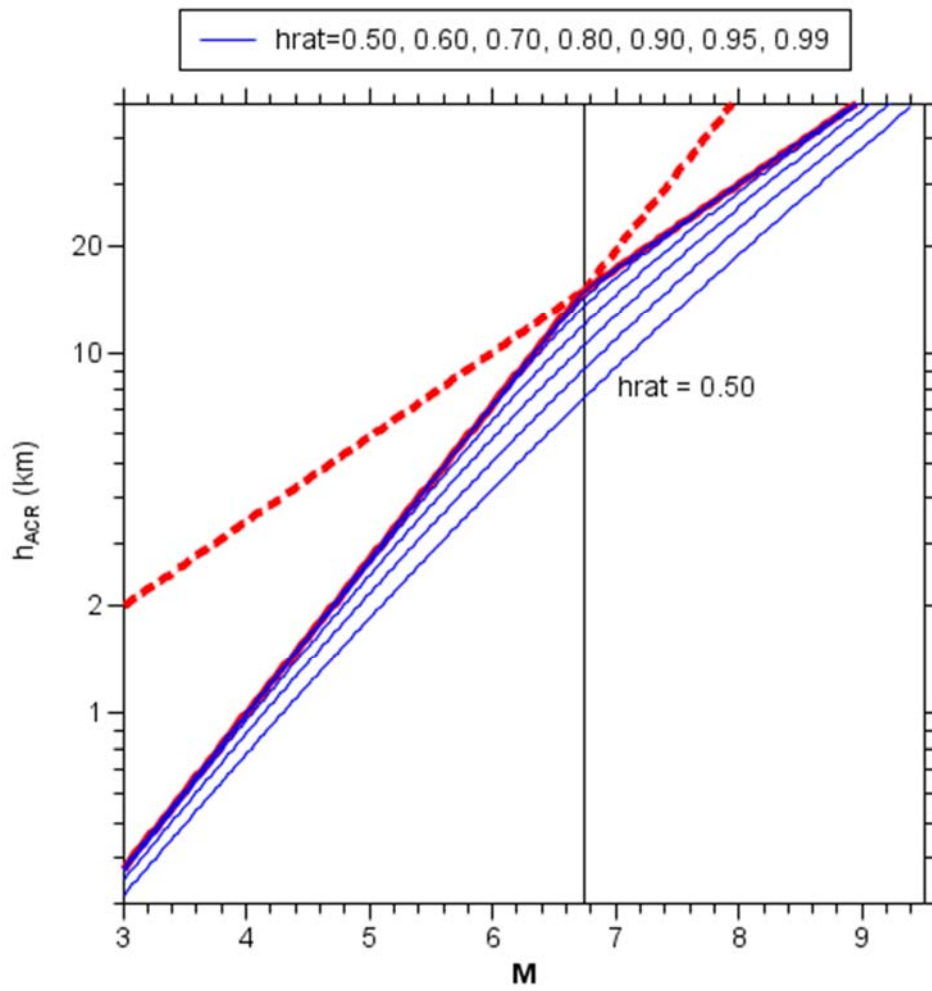$$\xi = \frac{(c_4 - c_2)\log 2}{\log h_{RAT}} \tag{8}$$

Note that the equations can be written in terms of natural logs by substituting "ln" for "log" and "e" for "10", and the coefficients for the two lines must be changed accordingly (the base 10 coefficients must be multiplied by ln(10)=2.303 if equations (2) and (3) are in terms of natural logs (base e)).

The above gives general equations, but for equivalence to BT15 and for the examples before showing that equivalence, I use the BT15 coefficients for both lines, as given in Table 1.

Table 1. The coefficients of the line segments used in BT15.

log base	c1	c2	c3	c4
10	-1.720	0.430	-0.405	0.235
e	-3.960	0.990	-0.933	0.541

To illustrate the flexibility of the single function to capture the two line segments with a smooth transition between them, Figure 1 shows the finite-fault factor for active crustal regions for the two line segments (in red, dashed beyond the intersection of the lines) and h for a suite of h_{RAT}.

Figure 1.

More detail in the transition region is given in Figure 2.

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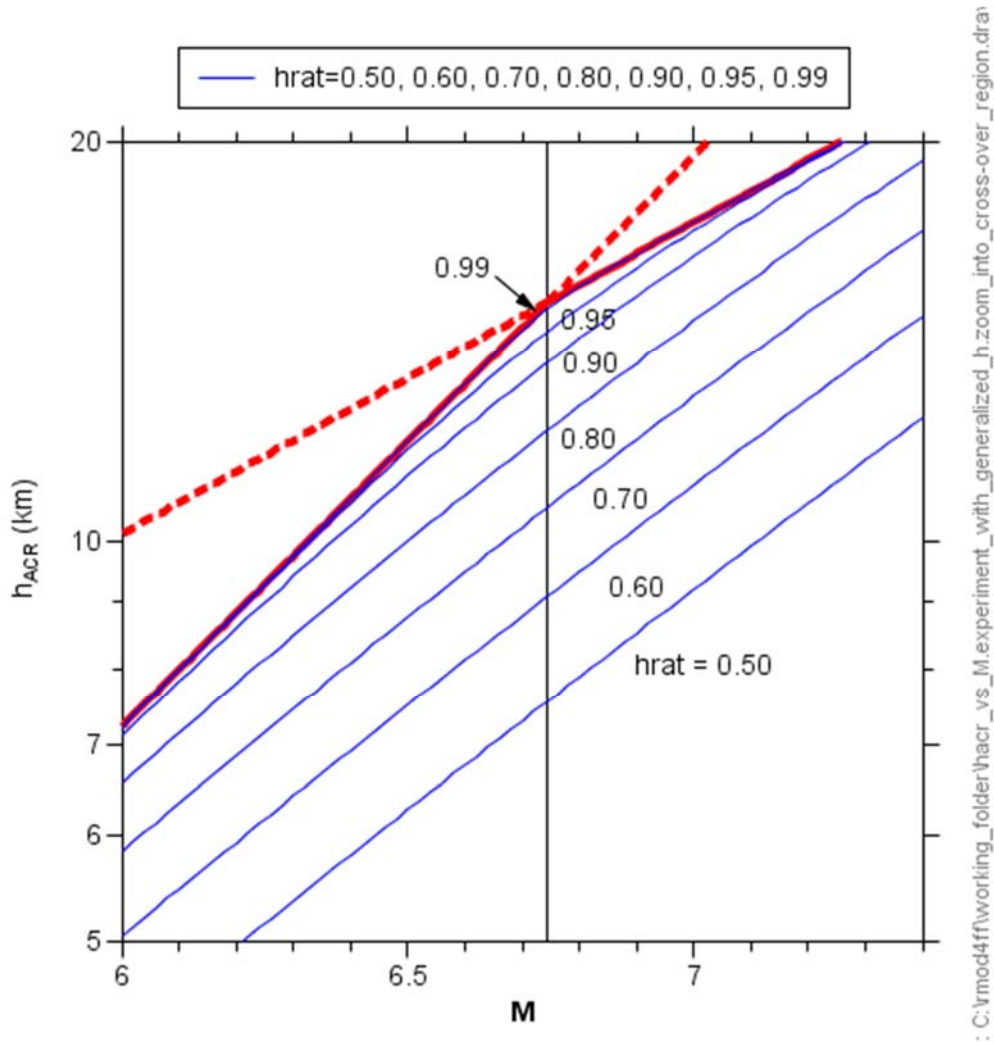*.application_to_BT15_FFF.v1.docx*

Figure 2.

Note that $h_{RAT} = 0.99$ gives a very close approximation to the two line segments.

Figure 3 shows that the function also works when the relative slopes of the two lines are reversed (as mentioned before, $h_{RAT} > 1.0$ in this case).
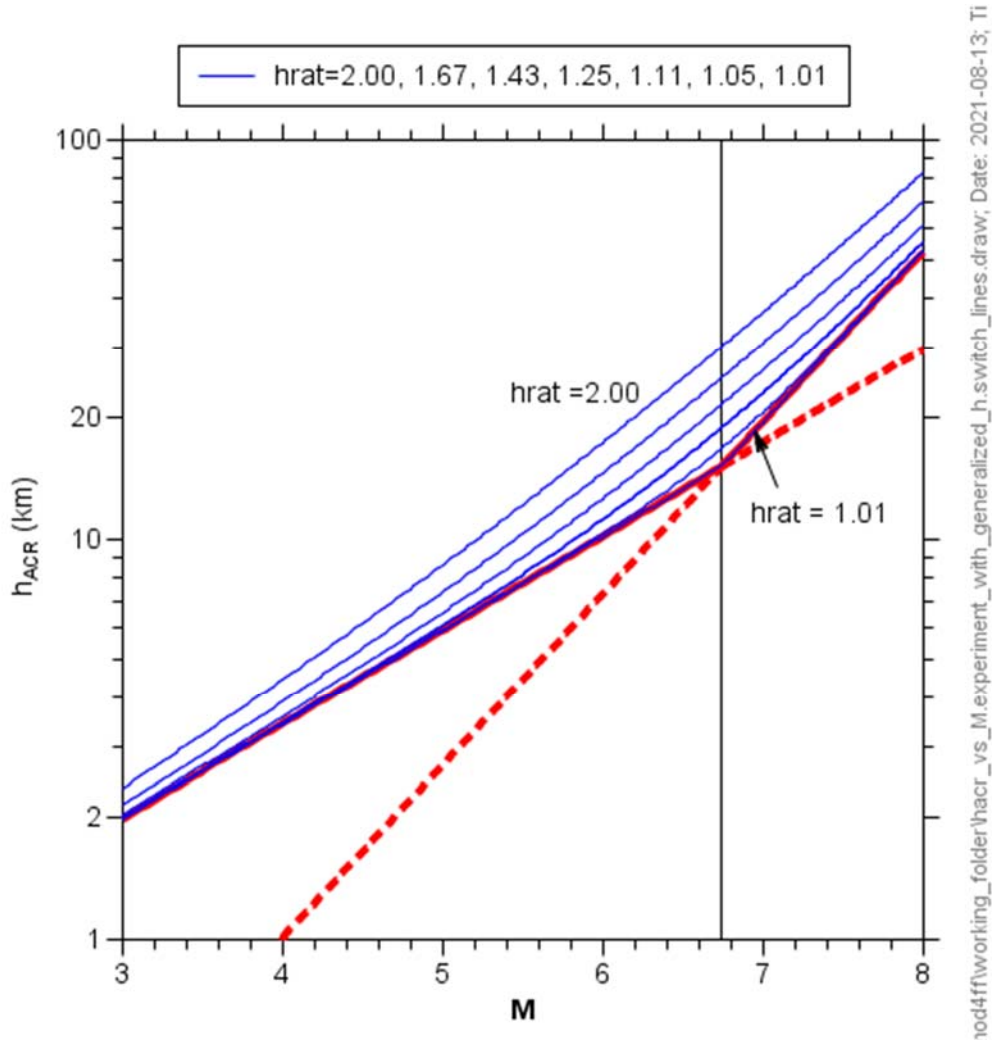
Figure 3.

I now turn to the coefficient h_{RAT} of the single equation that will produce a close approximation to BT15. With the coefficients of the lines in Table 1, the transition magnitude is

$$\mathbf{M}_T = 6.74359 \tag{9}$$

I initially chose $h_{RAT} = 0.8939$ based on the ratio of the BT15 smooth h at the transition magnitude to the h value of the individual lines at the transition (cross-over) magnitude. This gave the following revision of Figure 1 in BT15:
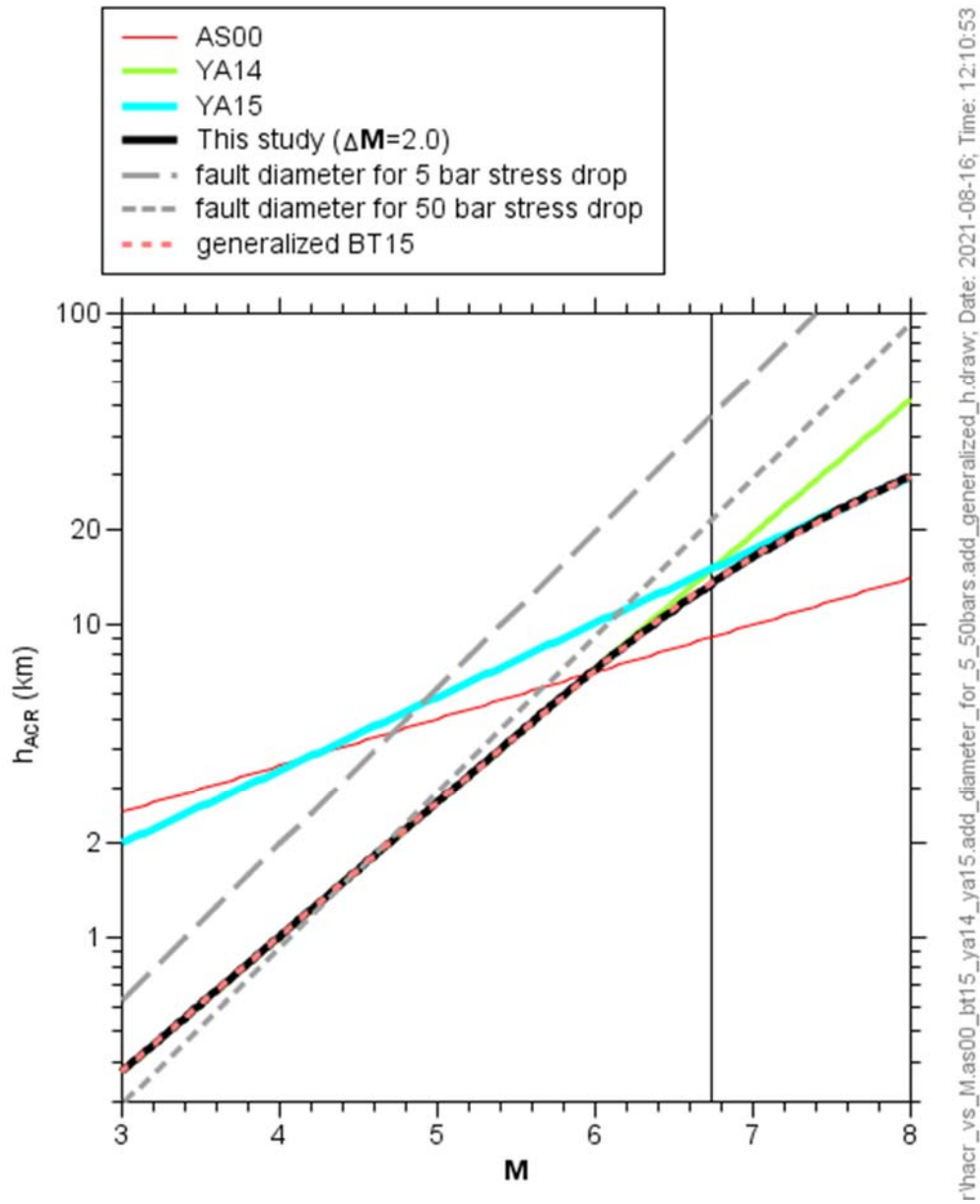
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.application_to_BT15_FFF.v1.docx

Figure 4.

The single equation for *h* given by equation (4) is shown by the dashed light red line. It is virtually indistinguishable from the BT15 line (solid black). To see the comparison in more detail, Figure 5 shows the ratio of the new *h* and the BT15 *h*, as a function of magnitude:

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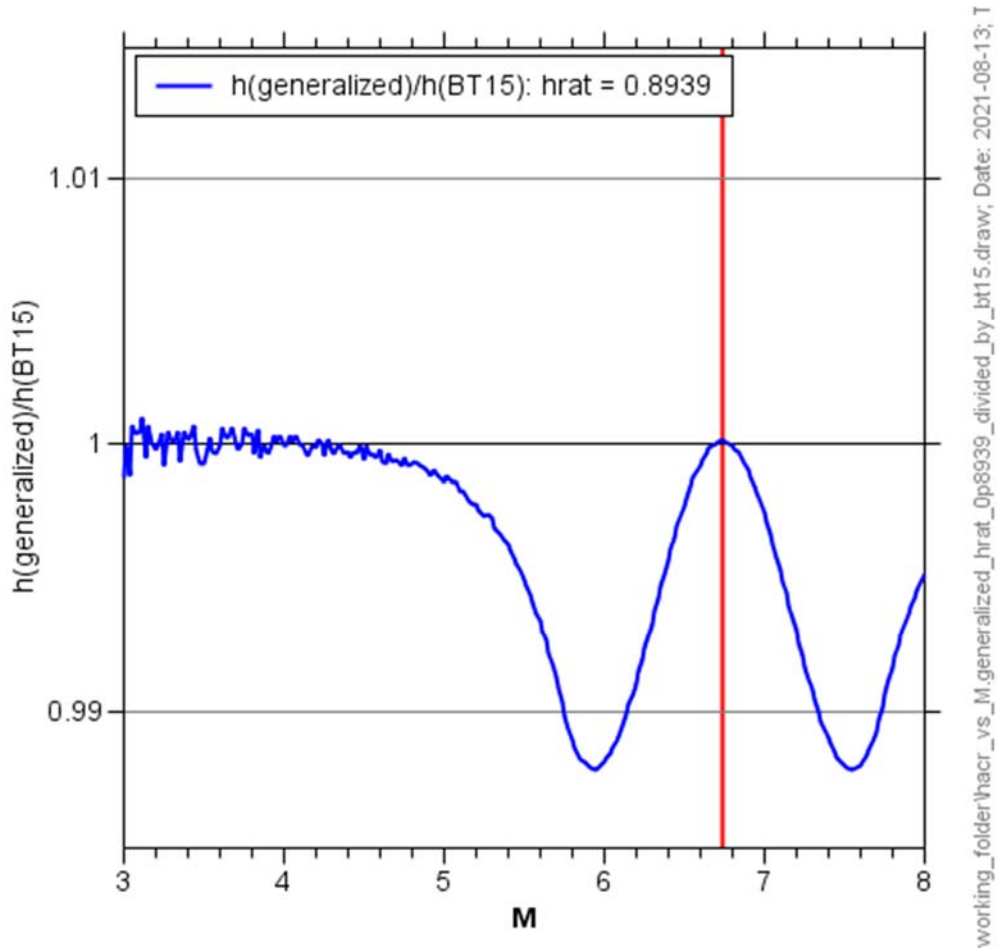*.application_to_BT15_FFF.v1.docx*

Figure 5. Using $h_{RAT} = 0.8939$.

By design the ratio is unity at the transition magnitude (that magnitude is indicated by the vertical red line), but the ratio shows that the approximation of the single function to BT15 is unbalanced. By using different values of h_{RAT} I found a value that balanced the two troughs in the ratio on either side of the transition magnitude and the value of ratio at the transition magnitude. This revised value of h_{RAT} was $h_{RAT} = 0.9015$, giving the following ratio as a function of magnitude (Figure 6).
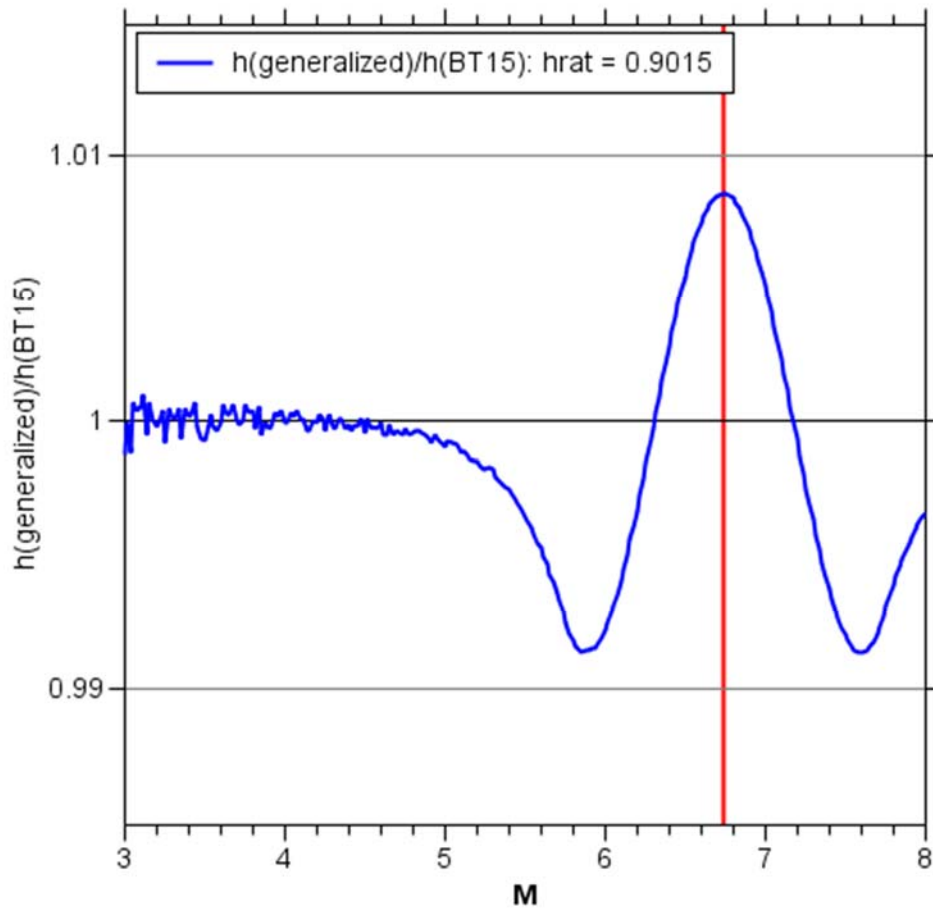
Figure 6. Using $h_{RAT} = 0.9015$.

Figure 7 shows the transition region in detail.
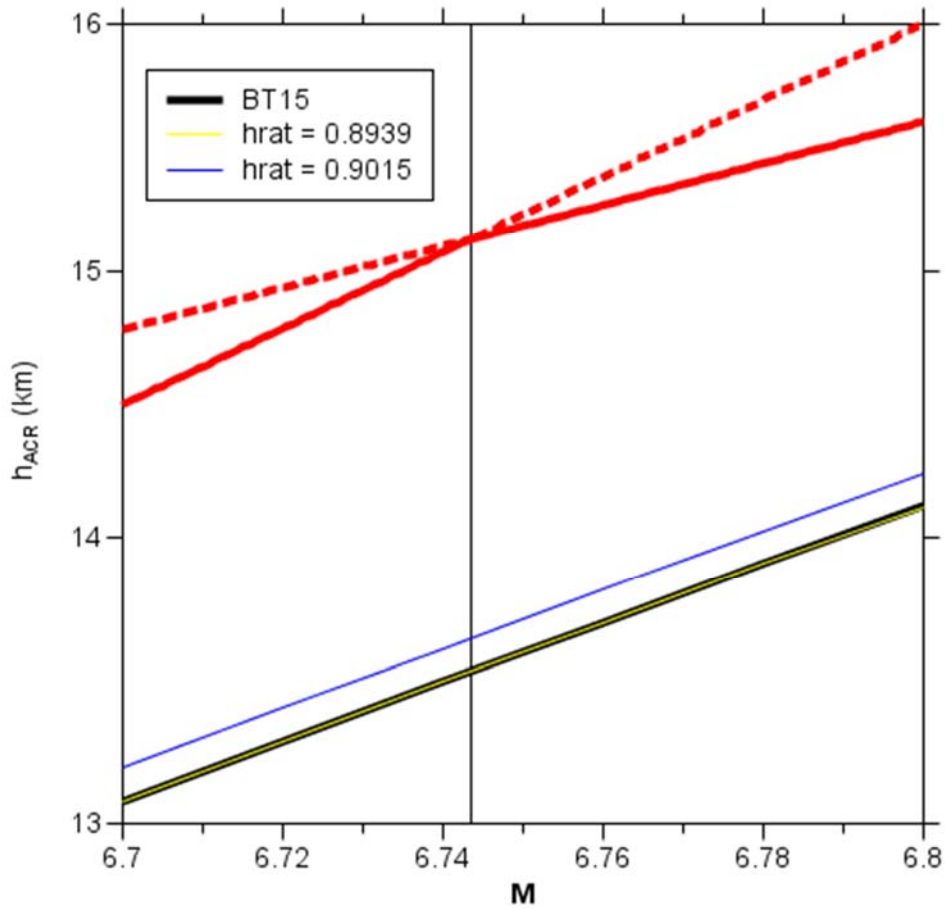
Figure 7.

In practice either value of h_{RAT} would give results using the single equation given in these notes equivalent to those using the tripartite BT15 equations, as shown in Figure 8.
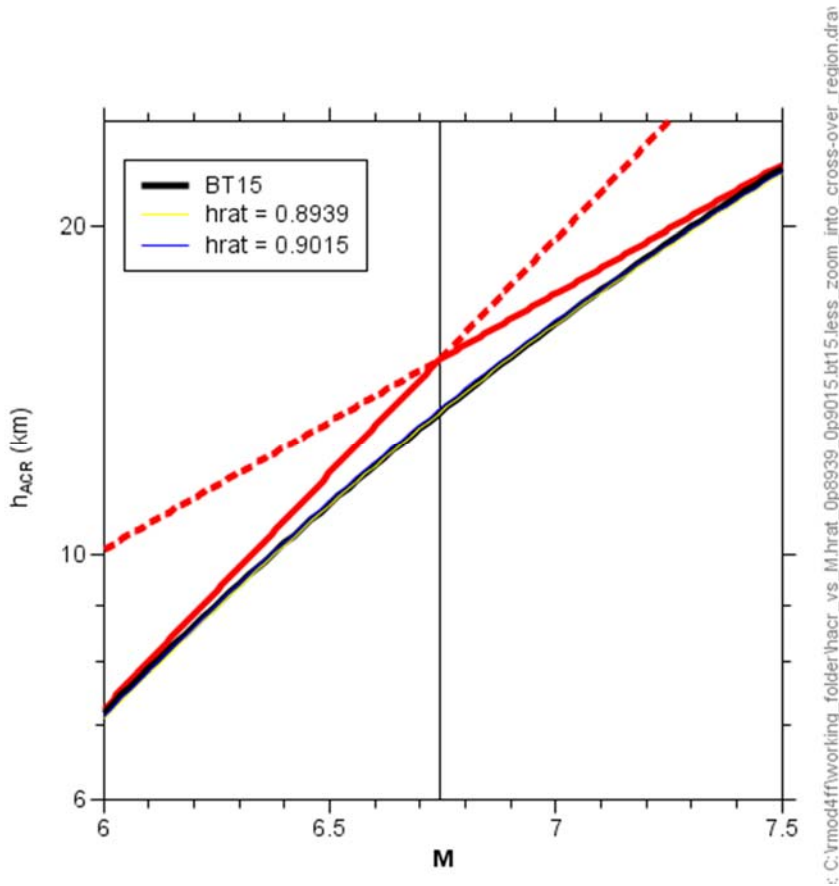
Figure 8.

In summary, here are the coefficients for the single equation that is equivalent to the BT15 equations for active crustal regions (ACR):

Table 2. Coefficients of single equation giving finite-fault factors equivalent to the BT15 tripartite equation for active crustal regions.

log base	c1	c2	c3	c4	hrat
10	-1.720	0.430	-0.405	0.235	0.9015
e	-3.960	0.990	-0.933	0.541	0.9015

Reference

Boore, D. M. and E. M. Thompson (2015). Revisions to some parameters used in stochastic-method simulations of ground motion, *Bull. Seismol. Soc. Am.* **105**, 1029–1041.

Stafford, P. J., D. M. Boore, R. R. Youngs, and J. J. Bommer (202x). Host-region parameters for an adjustable model for crustal earthquakes: facilitating the implementation of the backbone approach to building ground-motion logic trees in PSHA, *Earthquake Spectra*, submitted.

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