Dependence of GMRotI50 on Tmax4Penalty for the penalty function: Recommend use RotD50 rather than GMRotI50

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Last year Norm Abrahamson suggested a new measure of ground motion that does not involve geometric means. I investigated this suggestion in a soon-to-be-published paper (Boore, 2010b). To be consistent with the GMRotInn and GMRotDnn notation, I coined the terms "RotInn" and "RotDnn", where "D" and "I" mean that the rotation angle used for the response spectra are ***d***ependent and ***i***ndependent of period, for the new measures. The period-independent-rotation-angle measures (including GMRotI50, the measure used for the first round of the NGA-West ground-motion prediction equations) require the construction of penalty functions, for which the maximum period in the calculation of the penalty functions are needed (call them Tmax4Penalty). As Boore *et al.* (2006) showed, GMRotI50 can be sensitive to the choice of the maximum period used in computing Tmax4Penalty. Two assumptions, either made implicitly or explicitly, have apparently been used in computing GMRotI50 for the NGA flatfile:

1) Tmax4Penalty = min(Tmax4PSA, Tusable (the maximum usable response spectral period, as determined from the low-cut filter corner frequencies))

2) Tmax4Penalty = Tmax4PSA, regardless of the low-cut filter corner frequencies.

Tmax4PSA is the maximum period for which measure of seismic intensity is computed (10 s for NGA-West1).

The main purpose of these notes is to investigate the sensitivity of GMRotI50 to Tmax4Penalty, using most of the data in the NGA flatfile. To do this, GMRotI50 is computed for the two ways of obtaining Tmax4Penalty, and the ratios of these two values of GMRotI50 are studied. The distributions of the ratios of the two estimates of GMRotI50 are not normal, but are strongly peaked near unity, with the ratios from a small number of the records falling above and below unity. For about 90% of the records GMRotI50 computed using the two choices of Tmax4Penalty are within about 10% of one another; the other 10% of the records are within 20 to 40% of one another (statistical tests suggest that these records are outliers for some reason, but I have not had time to study them individually). The range spanned by the outliers is larger for periods in the 0.2 to 2 s range than for shorter or longer periods.

Although the choice of Tmax4Penalty makes little overall difference in GMRotI50, I recommend that the current NGA-West and NGA-East projects use the RotD50 measure (and if absolutely required by engineers, RotD100). Advantages of the period-dependent-rotation-angle RotDnn measures include: 1) no penalty function needs to be computed and 2) RotD50 and RotD100 are both obtained from the same distribution, with no geometric means being used---the procedure for obtaining the ground-motiom measures is easier to describe than GMRotI50 and is aesthetically and logically more satisfying than using different procedures for the median and maximum measures of seismic intensity.

Note that the period-independent-rotation-angle RotInn measures are definitely not recommended. As I show in Boore (2010b), RotI100 does not correspond to the true spectral maximum at all periods (as it is computed from the response spectra for a single rotation angle) whereas RotD100 is by definition the maximum response spectrum over all rotation angles.

Here is a figure from Boore (2010b) showing the ratio of RotD50 to GMRotI50 (RotI50 to GMRotI50):

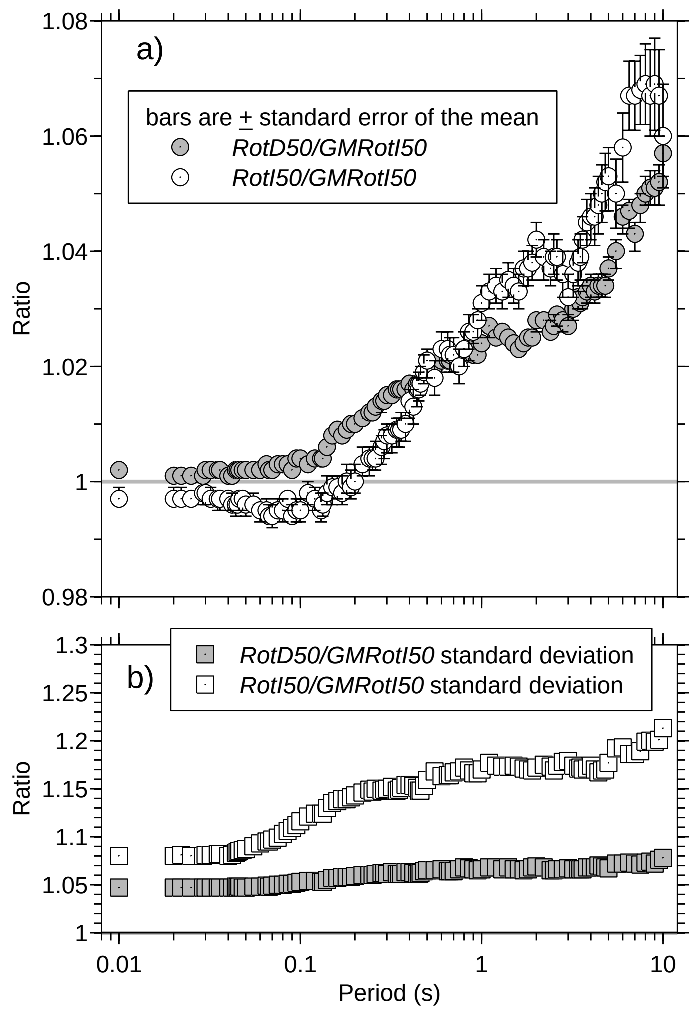

Figure 1. The antilogarithm of the mean and standard deviation of the logarithm of *RotD50/GMRotI50* and *RotI50/GMRotI50* for individual events as a function of oscillator period (from Boore, 2010b).

The program for computing the various measures is smc2psa_rot_gmrot, available from the online publications page on my web site (www.daveboore.com). In that program I have an option for Tmax4Penalty to be computed as either the maximum period for which response spectra are to be computed, or as 1/max(flc1, fc2), where flc1 and flc2 are the low-cut filter corner frequencies for horizontal components 1 and 2. I allow an option to multiply 1/max(flc1, fc2) by a factor, such as ½ or 1/1.25, to obtain the maximum usable period (see lowest_usable_freq_for_response_spectra_v20.pdf in the "daves notes" page on www.daveboore.com or Akkar and Bommer (2006)). Originally I as going to include the ability to read time series in either smc format or pea form (used for the PEER NGA project), and in the latter case extract the low-cut filter corner frequency from the header lines of the PEA files. Unfortunately, the low-cut (HP) filter frequencieis are not in a consistent place in the headers, sometimes starting at column 60, other times at column 59. In some cases the filter frequencies are missing, and in others the word "unknown" is used in the place of the filter frequency. I'm not a programmer for the PEER project and will let others adapt my programs for use in the NGA-West and NGA-East projects, if desired.

I computed GMRotI50 using the two options given above (for full set of 105 NGA periods (Chiou *et al.*, 2008), and the record selection criteria of Boore and Atkinson, 2008, except that aftershock records are allowed; this resulted in 3225 two-component records being used. Unlike the program smc2psa_rot_gmrot, just described, I used a more specialized program (nga2psa_rot_gmrot) that read input from the NGA flatfile and used time series in the NGA format (the time series are in the PEER NGA Master_TH_Base on the CD distributed to the NGA developers). The program used flowusable from the NGA flatfile, so that factors have been applied to max(flc1, flc2).

(some details for me: the computations were done in \rot_gmrot\working, using the programs nga2psa_gmroti_only.for and nga2psa_gmroti_only.nper_rec.from.thigh.for; these programs are a simplification of nga2smc_rot_gmrot, which reads the NGA time series, but I am not ready to distribute this program because it needs to be updated to be equivalent to the recently revised smc2psa_rot_gmrot program).

Before showing plots of the ratios of GMRotI50 computed using the two measures of Tmax4Penalty, I show below a plot of the distribution of Tusable for the records in the NGA flatfile.

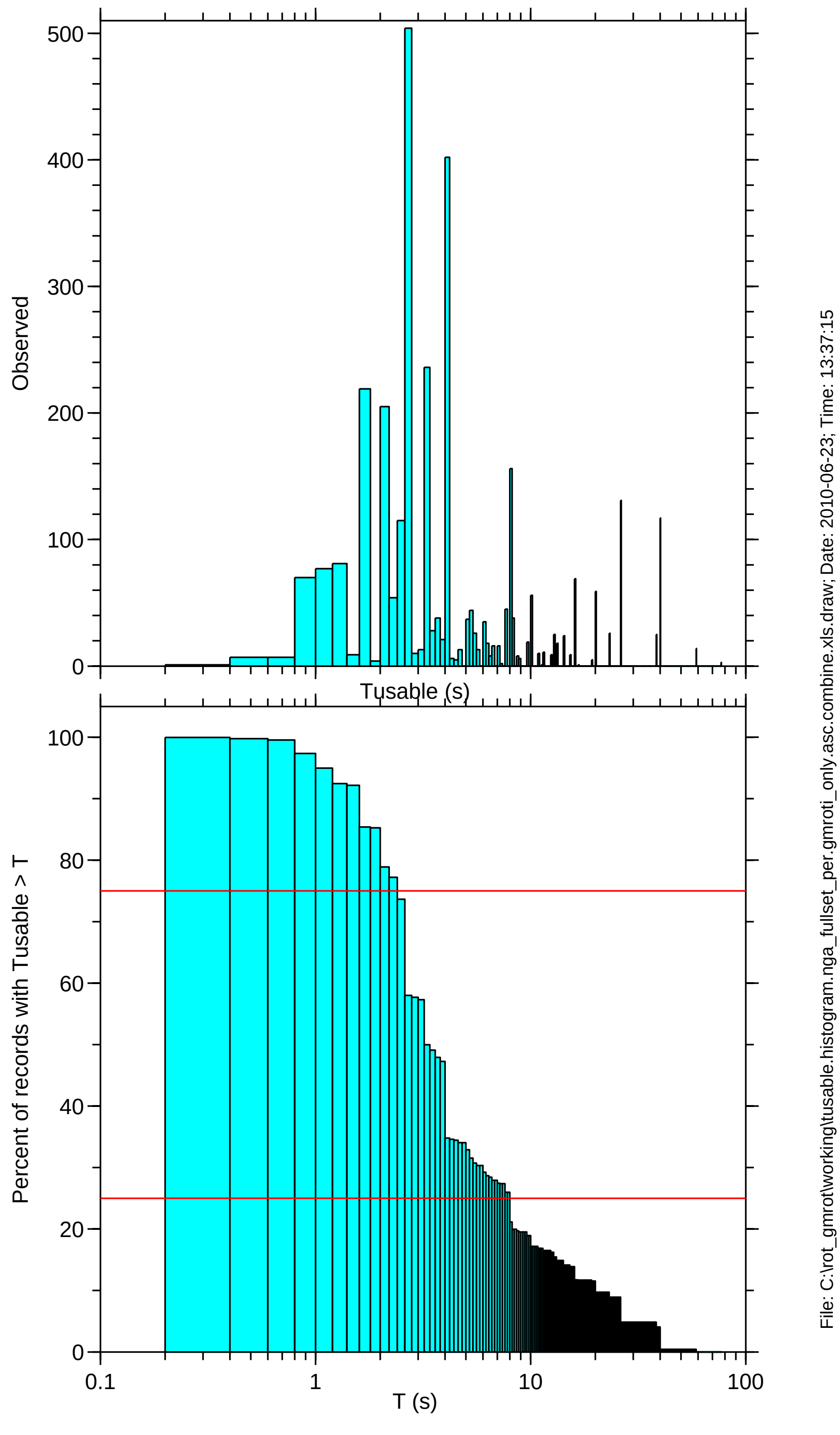

Figure 2. Density distribution (top) and complementary cumulative distribution (below) of Tusable. The red horizontal lines in the bottom figure are plotted at 25% and 75% values. These show that half of the records have Tusable between about 2.3 and 7.8 s.

The bottom graph in Figure 2 may be the most useful, as it can be used to obtain the percent of records for which GMRotI50 can be computed for a given period T. For example, 100% of the records will provide a value at T=0.2 s, but only about 75% and 25% of the records will provide values for T equal to 2.3 s and 7.8 s, respectively.

Here are plots showing the ratio of GMRotI50 computed using Tusable for Tmax4Penalty to that using the maximum spectral period of 10 s, vs. record index number (I only show ratios for periods less than Tusable = 1/flowusable, which has been taken from the NGA flatfile, which means that fewer points are plotted for the long period than for the short periods).

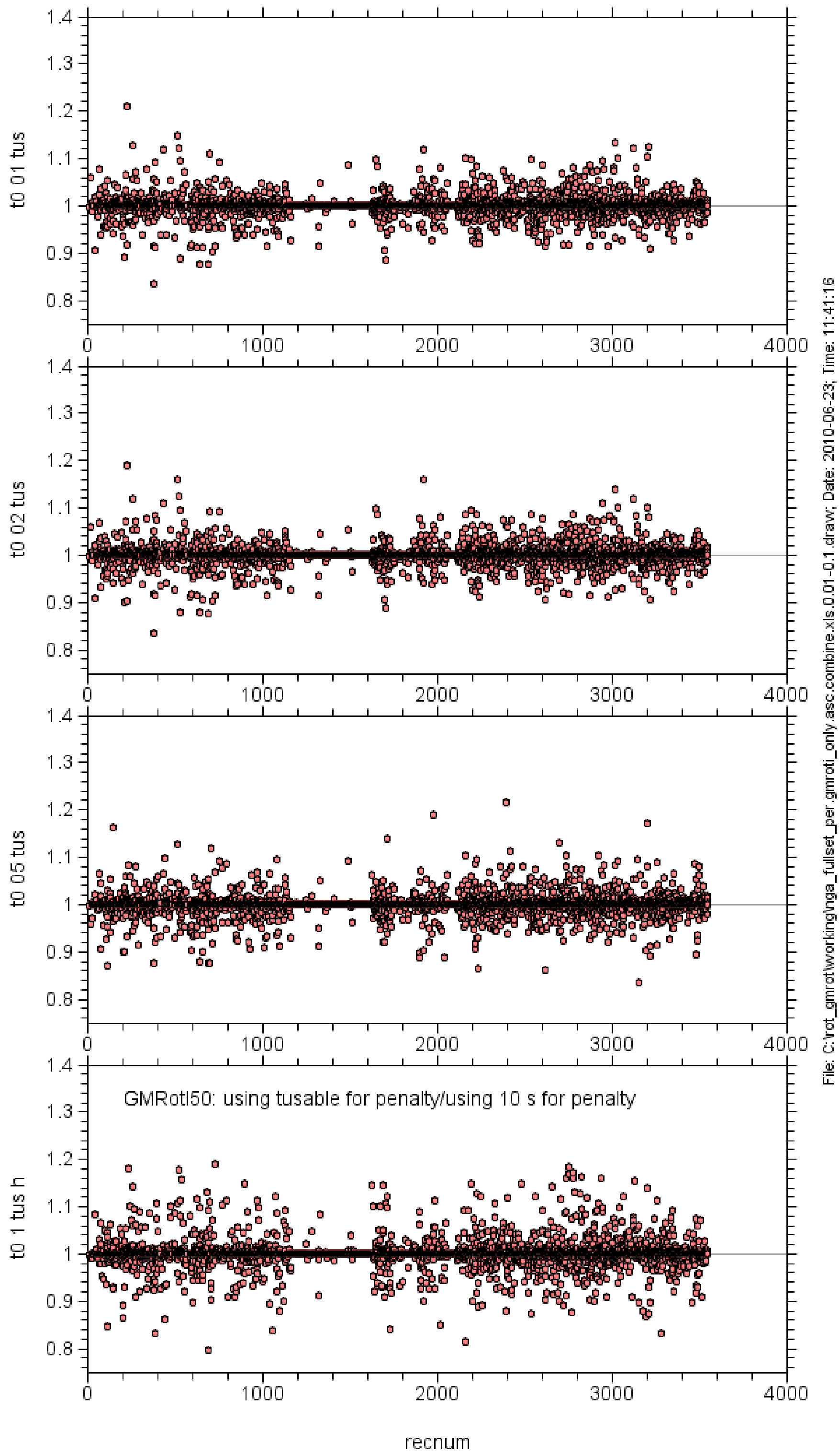

Figure 3. ratio of GMRotI50 computed using flowusable for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 0.01, 0.02, 0.05, and 0.1 (top to bottom; the period is shown on the ordinate title).

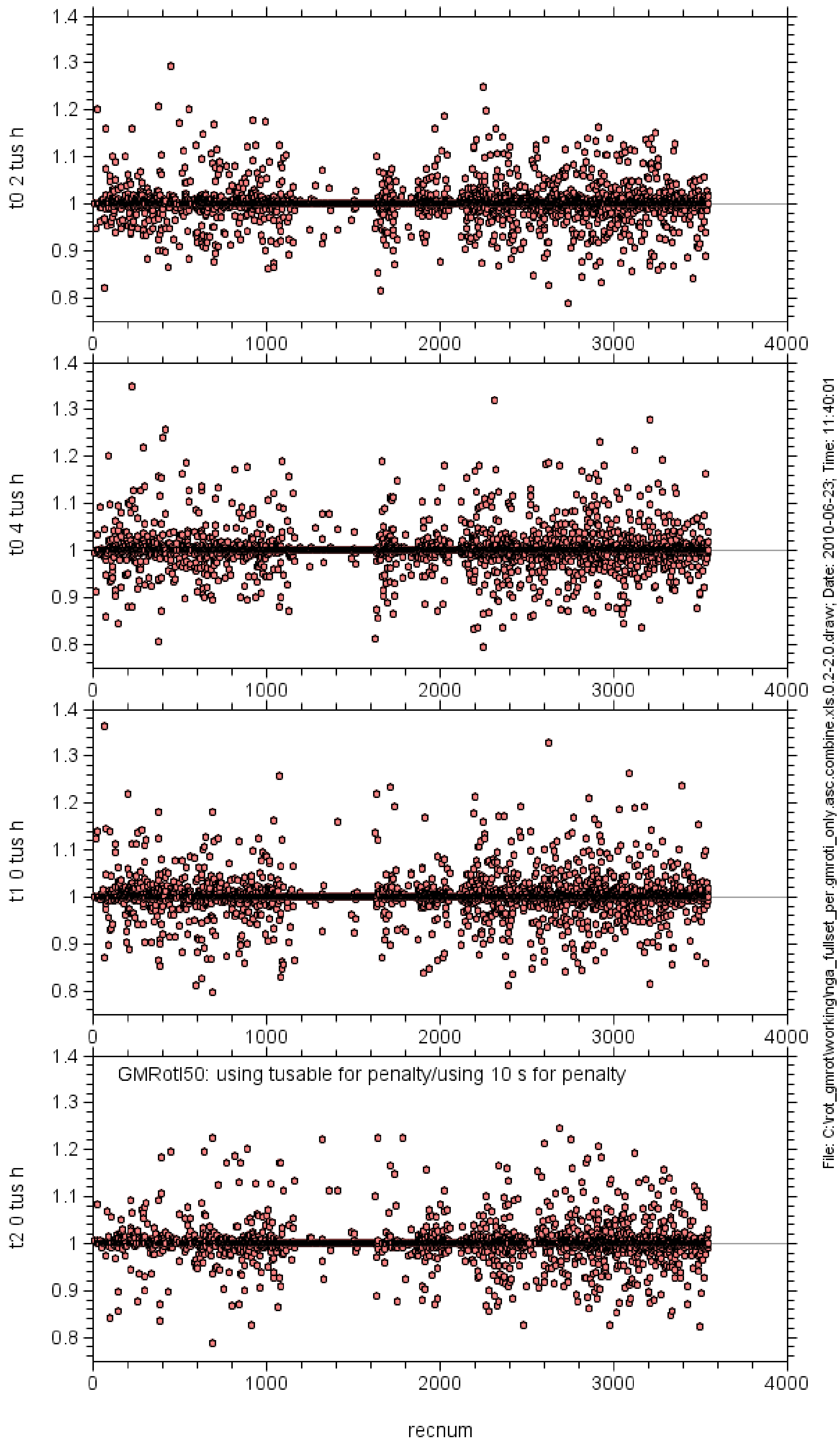

Figure 4. ratio of GMRotI50 computed using flowusable for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 0.2, 0.4, 1.0, and 2.0 (top to bottom; the period is shown on the ordinate title).

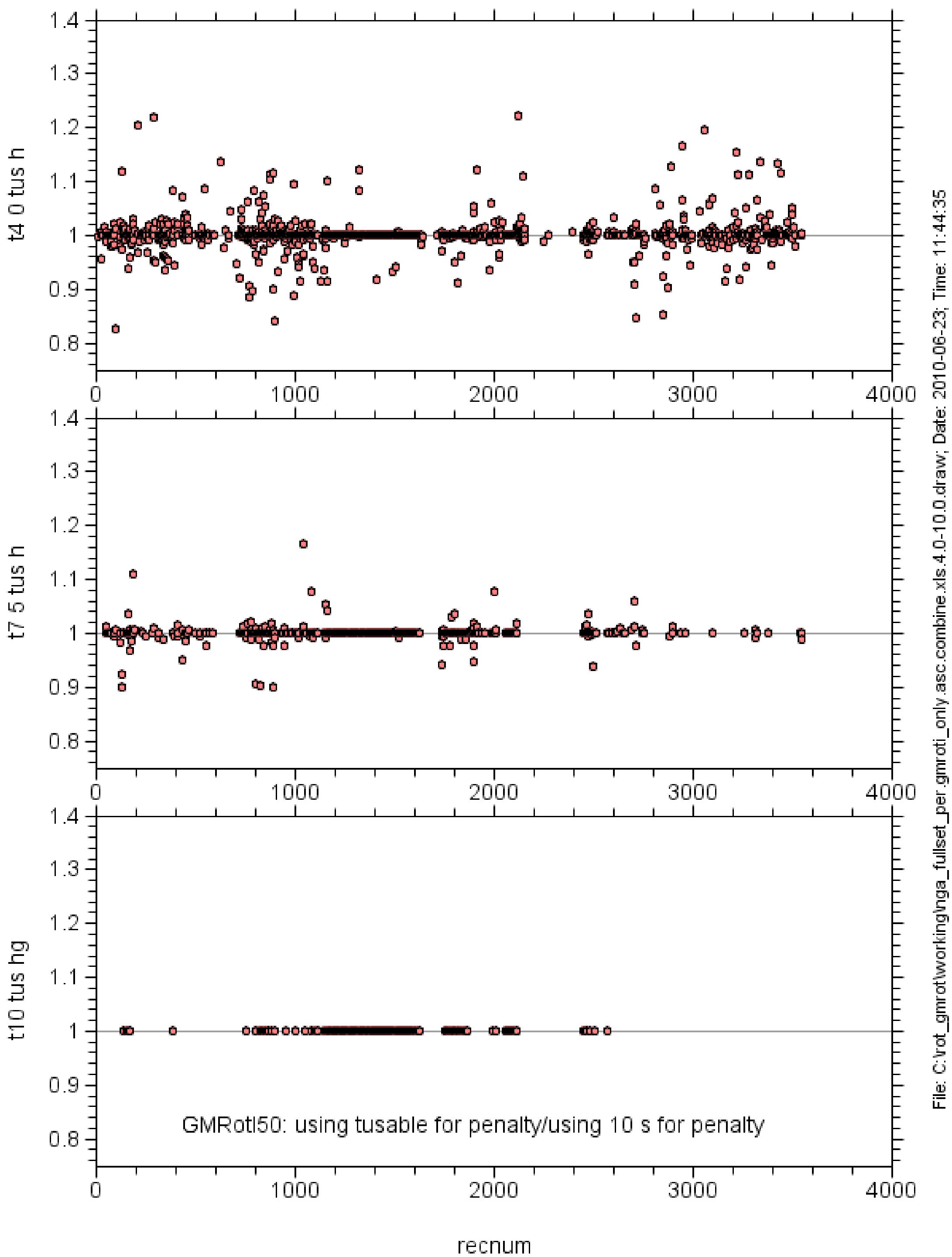

Figure 5. ratio of GMRotI50 computed using flowusable for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 4.0, 7.5, and 10.0 (top to bottom; the period is shown on the ordinate title).

Here is another set of plots, with the ratios plotted against Tusable.

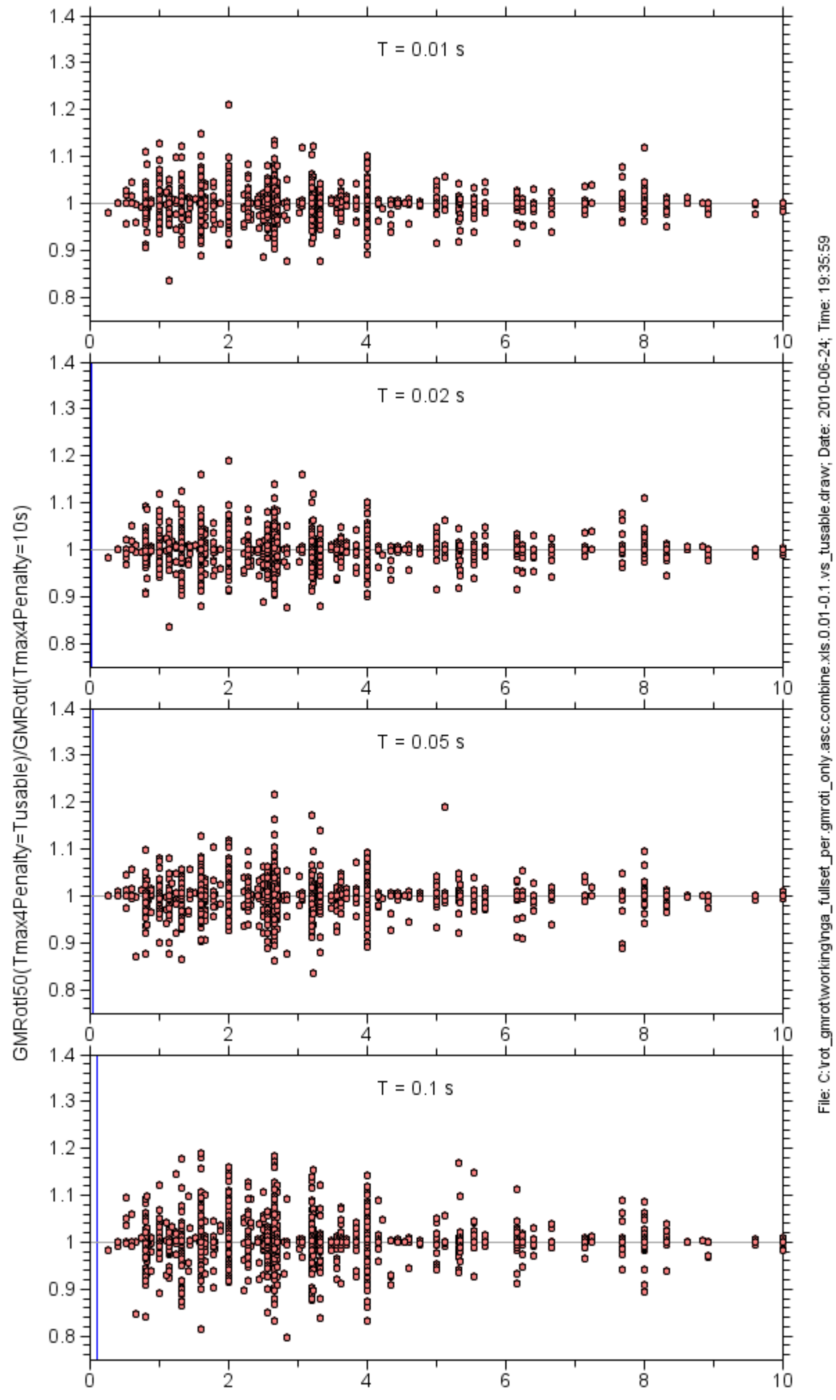

Figure 6. Ratio of GMRotI50 computed using Tusable (=1/flowusable) for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 0.01, 0.02, 0.05, and 0.1 (top to bottom; the period is shown on the ordinate title). The blue vertical lines are plotted at T4GMRotI50 (the period for the computed values of GMRotI50; e.g., for the bottom graph this is 0.1 s); no values of the ratio are plotted for Tusable < T4GMRotI50.

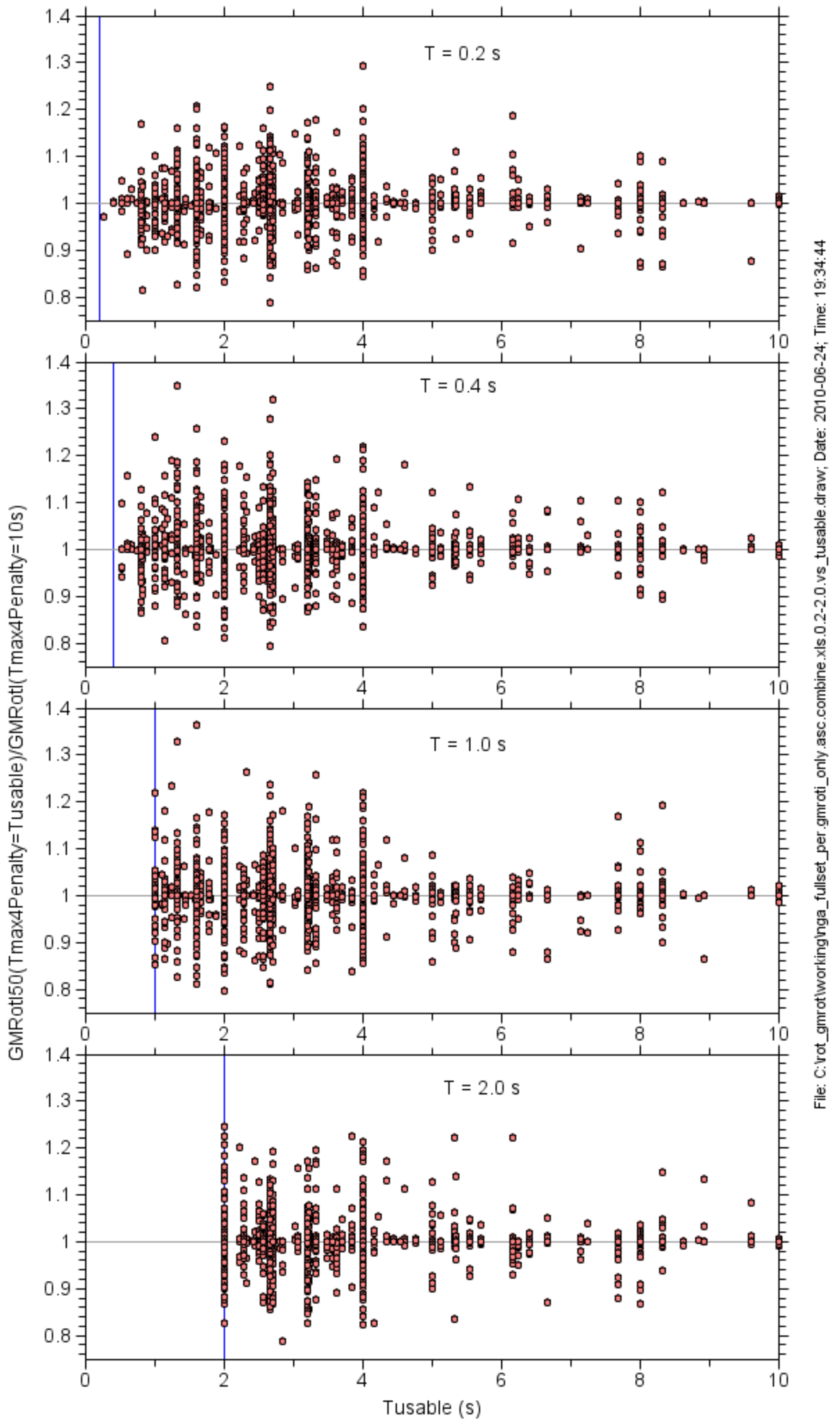

Figure 7. Ratio of GMRotI50 computed using Tusable (=1/flowusable) for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 0.2, 0.4, 1.0, and 2.0 (top to bottom; the period is shown on the ordinate title). The blue vertical lines are plotted at T4GMRotI50 (the period for the computed values of GMRotI50; e.g., for the bottom graph this is 2.0 s); no values of the ratio are plotted for Tusable < T4GMRotI50.

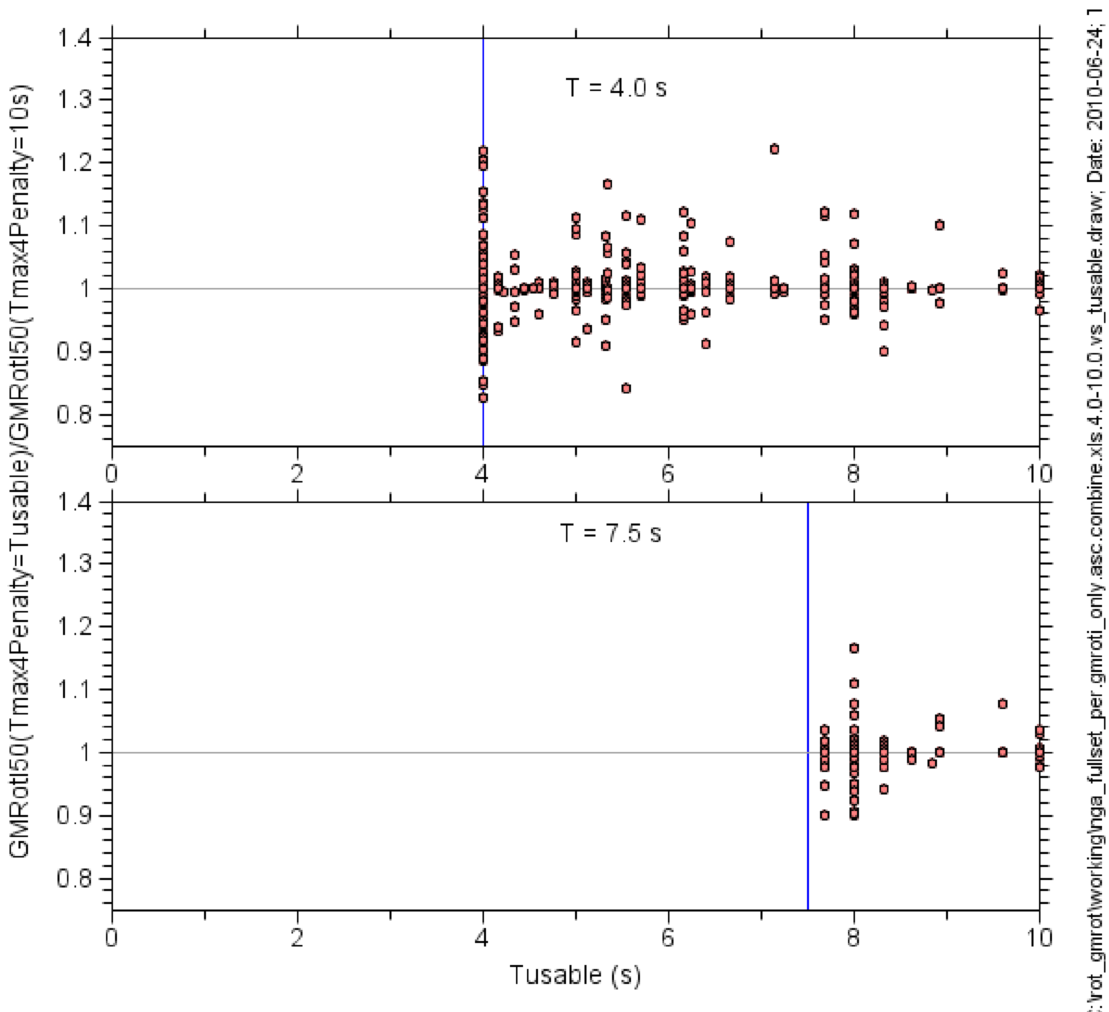

Figure 8. Ratio of GMRotI50 computed using Tusable (=1/flowusable) for Tmax4Penalty to that using 10 s for Tmax4Penalty for periods of 4.0 s and 7.5 s (top to bottom; the period is shown on the ordinate title). The blue vertical lines are plotted at T4GMRotI50 (the period for the computed values of GMRotI50; e.g., for the bottom graph this is 7.5 s); no values of the ratio are plotted for Tusable < T4GMRotI50. There is no graph for T=10 s because all ratios are unity (see Figure 5).

These plots are more useful than those in Figures 3, 4, and 5, because they give an idea of the relation of the outliers to Tusable: the outliers seem to span a wider range when Tusable is much smaller than 10 s, which is understandable. These plots are good for identifying outliers, but they give a misleading impression of the distribution of the ratios. This is better obtained from this series of plots (for the set of periods as in Figures 6, 7, and 8).

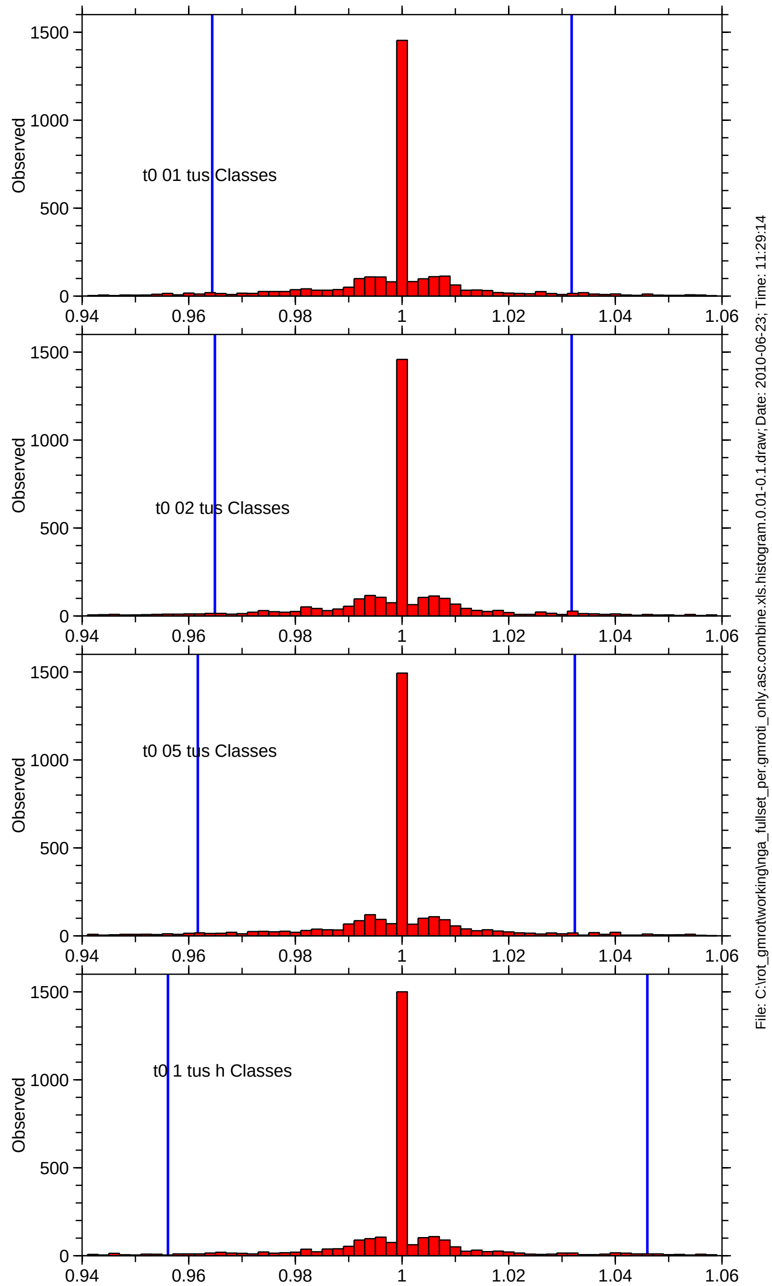

Figure 9. Histograms of ratio of GMRotI50 for periods of 0.01, 0.02, 0.05, and 0.1 (top to bottom; the period is shown on the ordinate title). The abscissa-axis range has been chosen to show some of the detail near the peak of the distribution, and as a result some of the outlying points shown in Figure 3 are not included. The blue vertical lines are at the 5% and 95% values of the distribution of the ratios.

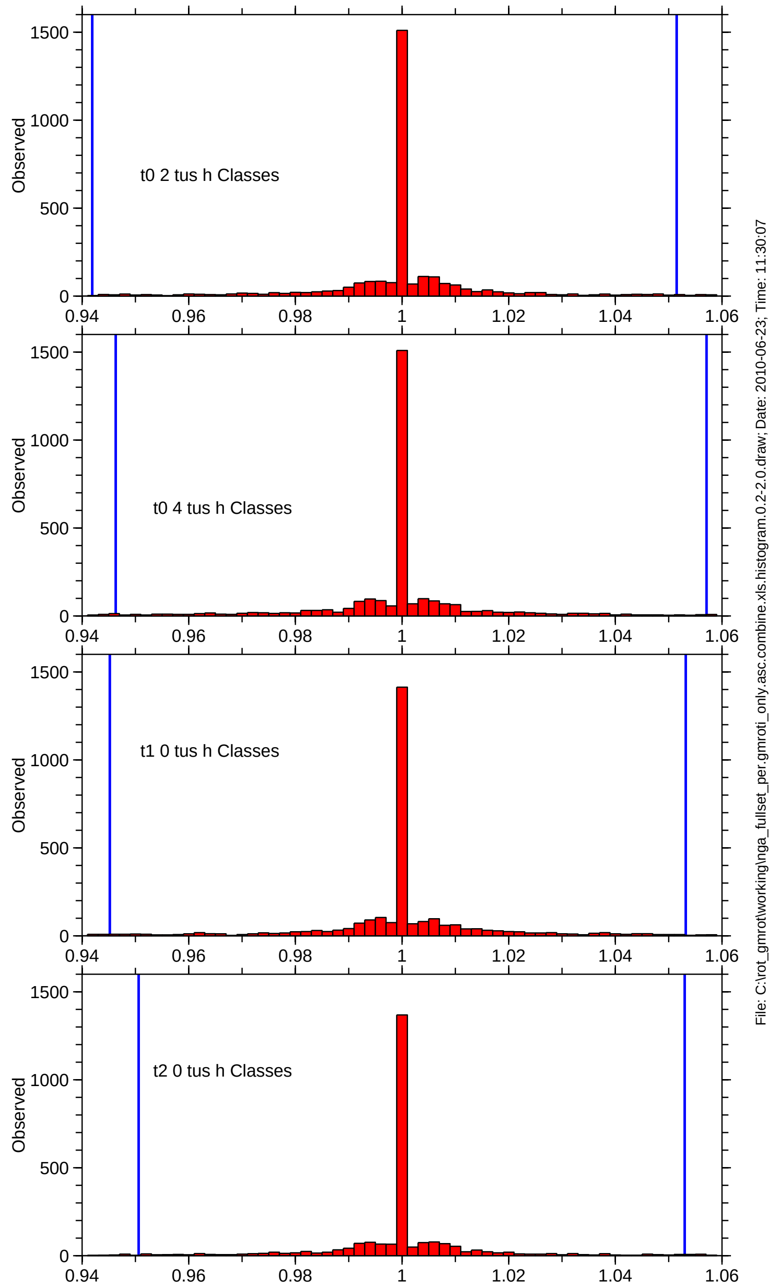

Figure 10. Histograms of ratio of GMRotI50 for periods of 0.2, 0.4, 1.0, and 2.0 (top to bottom; the period is shown on the ordinate title). The abscissa-axis range has been chosen to show some of the detail near the peak of the distribution, and as a result some of the outlying points shown in Figure 4 are not included. The blue vertical lines are at the 5% and 95% values of the distribution of the ratios.

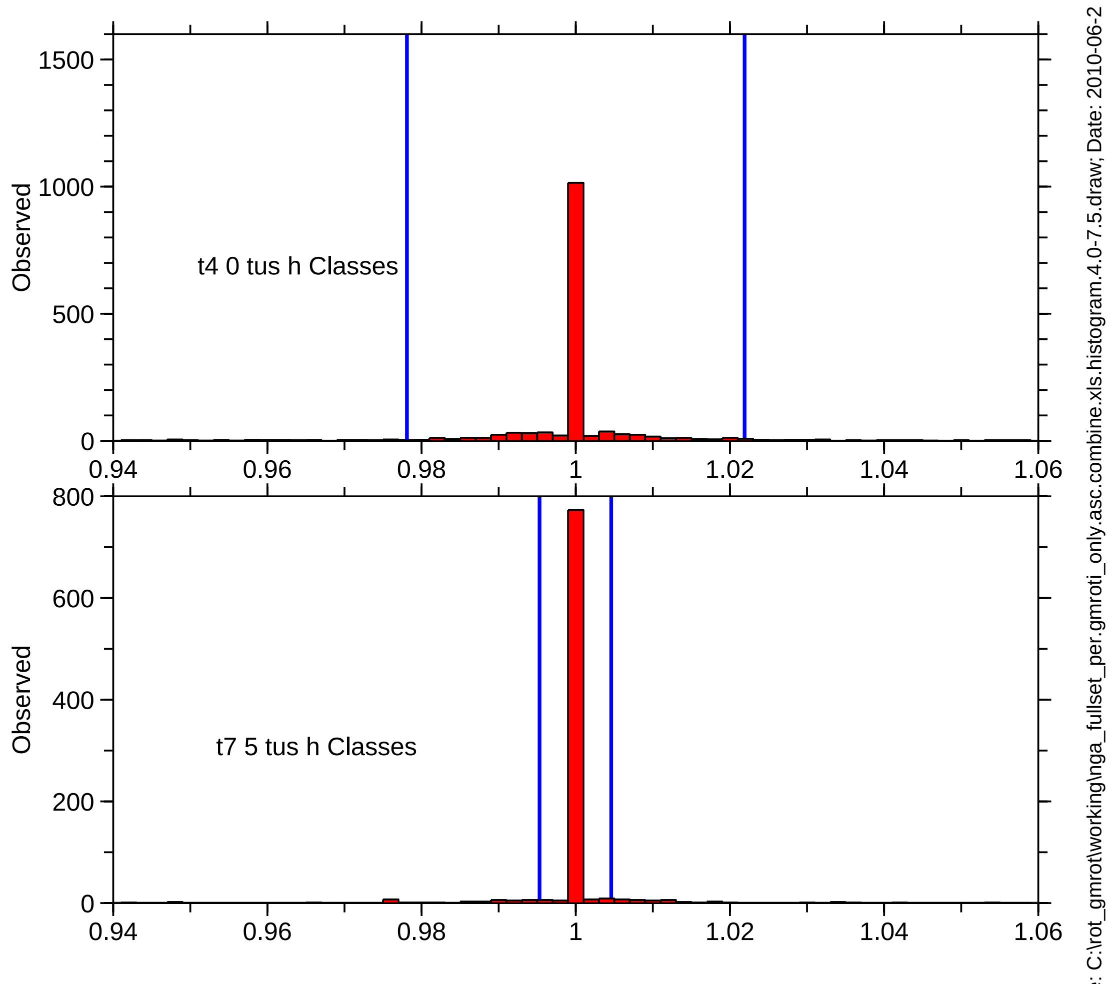

Figure 11. Histograms of ratio of GMRotI50 for periods of 4.0 and 7.5 (top to bottom; the period is shown on the ordinate title). The abscissa-axis range has been chosen to show some of the detail near the peak of the distribution, and as a result some of the outlying points shown in Figure 5 are not included. The distribution for T=10 s is not shown, as all values have a ratio of unity. The blue vertical lines are at the 5% and 95% values of the distribution of the ratios.

The plots suggest, and statistical tests confirm, that the ratios are not distributed normally. In addition, much of the scatter seen in Figures 3, 4, and 5 satisfy statistical tests for outliers.

Here is a table giving some statistical measures for the various periods, including quantiles (Q) and the range spanned by the 5% to 95% quantiles:

T(s)	Mean	StaDev	Min	Max	n	CoeVar%	Q0.05%	Q25%	Q50%	Q75%	Q0.95%	95%-05%
0.01	0.999	0.022	0.834	1.208	3225	2.17	0.964	0.996	1.000	1.002	1.032	0.07
0.02	0.999	0.022	0.833	1.189	3225	2.17	0.965	0.996	1.000	1.002	1.032	0.07
0.05	0.999	0.023	0.834	1.217	3225	2.35	0.962	0.996	1.000	1.001	1.032	0.07
0.10	1.000	0.030	0.795	1.188	3225	3.02	0.956	0.997	1.000	1.002	1.046	0.09
0.20	1.000	0.034	0.787	1.292	3225	3.44	0.942	0.998	1.000	1.003	1.051	0.11
0.40	1.001	0.038	0.793	1.347	3222	3.77	0.946	0.997	1.000	1.003	1.057	0.11
1.00	1.001	0.038	0.795	1.362	3140	3.83	0.945	0.997	1.000	1.004	1.053	0.11
2.00	1.001	0.036	0.788	1.245	2750	3.61	0.951	0.998	1.000	1.000	1.053	0.10
4.00	1.001	0.025	0.825	1.221	1526	2.51	0.978	1.000	1.000	1.000	1.022	0.04
7.50	1.000	0.012	0.900	1.164	885	1.23	0.995	1.000	1.000	1.000	1.005	0.01
10.00	1.000	0.000	1.000	1.000	557	0.00	1.000	1.000	1.000	1.000	1.000	0.00

References (all of my papers listed below are available from the online software link at http://www.daveboore.com)

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Chiou, B.S.J., Darragh, R., Gregor, N., and Silva, W. (2008). NGA project strong-motion database, *Earthquake Spectra* **24**, 23-44.