Some notes on using interpolation of two given velocity profiles to obtain a profile for a specified $\bar{V}(Z)$ (updated and generalized on 24 July 2015).

David M. Boore

The idea for using interpolation of two given velocity profiles to obtain a profile corresponding to a specified time-averaged velocity from the surface to depth Z ($\bar{V}(Z)$) came from Cotton et al. (2006) (Cea06). The two given profiles used by Cea06 were the Boore and Joyner (1997) (BJ97) generic rock and very hard rock shear-wave velocity profiles (Tables 1 and 2 in BJ97), but they could be any two profiles. Here is the relevant portion of Cea06, describing their interpolation method:

for the target region. We suggest to generate the model set such that for a V_{S30} of 620 m/s the model matches the Californian rock model (Boore and Joyner, 1997, Table 1) while for V_{S30} of 2800 m/s it matches the hard rock model for ENA (Boore and Joyner, 1997, Table 2). For each chosen V_{S30} rock velocity an interpolation fraction Ifrac$_{V_{S30}}$ is defined (in log scale) with respect to V_{S30} of the two Boore and Joyner (1997) rock models ($V_{S30} = 620$ and 2800 m/s):

$$\text{Ifrac}_{V_{S30}} = \frac{\log(V_{S30}) - \log(620)}{\log(2800) - \log(620)} \tag{1}$$

where V_{S30} is given in m/s

The generic models of Boore and Joyner (1997) are anchored at $z_a = 1, 30, 190, 4000$ and 8000 meters (Table 3). Our generic shear-wave velocities for a given interpolation fraction are then defined at these anchoring depths using the following formula:

$$\beta_{V_{S30}}(z_a) = 10^{(\text{Ifrac}_{V_{S30}} \cdot (\text{Log}(\beta_{za2}) - \text{Log}(\beta_{za1})) + \text{Log}(\beta_{za1}))} \tag{2}$$

where β_{za2} and β_{za1} are the values of Boore and Joyner (1997) rock models at the anchoring depths (Table 3).

The problem with this approach is that I do not see that it guarantees the target $\overline{V}(Z)$ (I assume that their method is applicable to any Z, not just 30 m, although the application shown later is for 30 m). I've developed an alternative approach based on the requirement that $\overline{V}(Z)$ is matched. Here is the development:

The definition of the time-averaged velocity to any depth Z is given by

$$\overline{V}(Z) = Z \bigg/ \int_0^Z \frac{1}{V(\xi)} d\xi . \tag{1}$$

I do not include a subscript "S" on $\overline{V}$, indicating shear-wave velocity, because the formulation works for any type of velocity profile. The notation for the average velocity in equation (1) is used for simplicity in later equations; in terms of the usual notation, $V_{S30} = \overline{V}(30)$, where now the velocity function in the integral in equation (1) is the shear-wave velocity. It is convenient to work with seismic slowness, which has a number of advantages over seismic velocity in site response studies, as discussed, e.g., by Brown et al. (2002) and Boore and Thompson (2007). The slowness and velocity are related to one another by

$$S(z) = 1 \big/ V(z) . \tag{2}$$

In terms of slowness, the equivalent of equation (1) is

$$\overline{S}(Z) = \frac{1}{Z} \int_0^Z S(\xi) d\xi , \tag{3}$$

and

$$\overline{V}(Z) = 1 \big/ \overline{S}(Z) . \tag{4}$$

Now assume that two slowness models are available, $S_1(z)$ and $S_2(z)$, and that a third profile is obtained from a linear combination of these two profiles, using the following equation:

$$S(z) = (1 - \beta) S_1(z) + \beta S_2(z) . \tag{5}$$

The coefficient β can be obtained by requiring that the average slowness to depth Z, $\overline{S}(Z)$, equals a desired value, $\overline{S}_D$. With this condition, equations (3) and (5) can be combined to give

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$$\overline{S}_D = (1-\beta)\overline{S}_1(Z) + \beta\overline{S}_2(Z), \qquad (6)$$

and this can be solved for β:

$$\beta = \frac{\overline{S}_D - \overline{S}_1(Z)}{\overline{S}_2(Z) - \overline{S}_1(Z)}. \qquad (7)$$

$S(z)$ can take on any value below depth Z without affecting the constraint that $\overline{S}(Z) = \overline{S}_D$, but for simplicity, we assume that equation (5) holds for all depths. In general, $S(z)$ becomes smaller with increasing depth and thus has proportionally less impact on site amplifications computed using the square-root impedance method than does $S(z)$ at shallow depths. For this reason differences in slowness profiles at deeper depths are less important than at shallow depths.

Working in the spreadsheet `C:\ena_hr_to_bc\bc_velocities_and_amps_cena\interpolate_bj97_hr_vhr_to_specified_vs30.xls`, I used both Cea06's and my interpolation methods to find the desired velocity profile (illustrated here for $V_{S30} = 760$ m/s and $V_{S30} = 1300$ m/s, using the BJ97 generic and very hard rock V_s profiles as profiles *1* and *2*, respectively). Note that for the Cea06 interpolation I had to make a small adjustment in the upper 1 m of the model to obtain the proper V_{s30}. In the travel time calculations I assumed straightline changes in velocity from one depth point to another (as this is what my program `site_amp` assumes). Down to 30 m I sampled the BJ97 generic and very hard rock velocities at 1 m spacing. Here is a table of V_{s30} for the models, when the target velocity to 30 m is 0.760 m/s:

model	V_{s30}
BJ97 generic rock (Table 1, BJ97)	0.618 km/s
BJ97 very hard rock (Table 2, BJ97)	2.780 km/s
Cotton et al (2006), no adjustment	0.765 km/s
Cotton et al (2006), adjusted layer 1 velocity	0.760 km/s
Equations (2), (5), and (7)	0.760 km/s

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And here is a plot of the profiles:
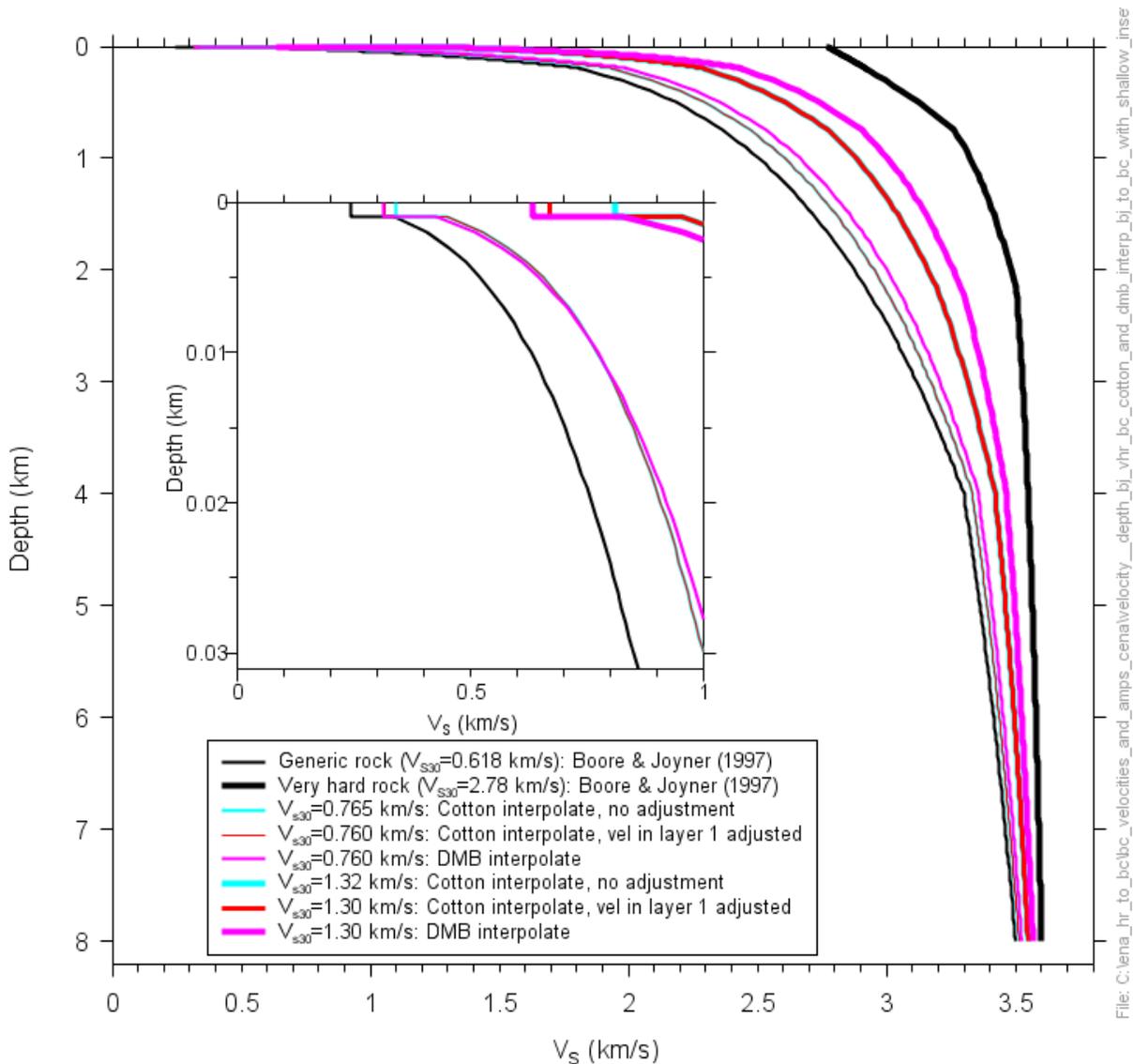
As shown above, only a small adjustment needs to be made to the velocity profile obtained by using the Cea06 interpolation, but the interpolation given by equations (2), (5), and (7) requires no adjustment (curves labeled "DMB", for my initials). The largest apparent difference in the velocity profiles derived from the two interpolation methods is at depths of one to several kilometers, but it is unlikely that these differences will lead to a significant difference in site amplification computed using these velocity profiles. Because it requires no adjustment, I prefer the interpolation method given by equations (2), (5), and (7).

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