Notes on relating density to velocity for use in site amplification calculations

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Calculations of site amplification require both velocity and density as a function of depth. But while the velocity model is usually specified, seldom is the density model given. I had built in a default relation between velocity and density in my program *site_amp* (in the SMSIM software package, available from the online software page of www.daveboore.com) but I was not happy with it (it is shown in Figure 4 below, and clearly my reservations were well founded), so these notes describe my replacement of that relation. These notes update a previous version by using many additional data to constrain better the relation between density and shear-wave velocity at low values of the shear-wave velocity.

Relating Density to Compressional-Wave Velocity:

A popular relation between density (ρ) and *P*-wave velocity (V_P) seems to be that of Gardner et al. (1974) (Gea74). The relation takes the following forms, depending on the units of V_P (in all cases the units of density are g/cm^3):

ft/s: $\rho = 0.23\,V_P^{\,0.25}$ (1)

km/s: $\rho = 1.74\,V_P^{\,0.25}$ (2)

m/s: $\rho = 0.31\,V_P^{\,0.25}$ (3)

Their relation is simply an approximate average of the relations for a number of sedimentary rock types, weighted toward shales. The relation comes from Figure 1 in Gea74.

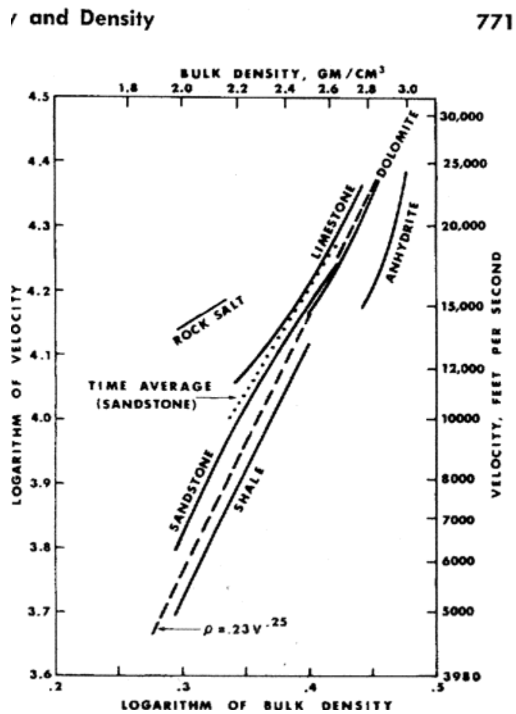

Figure 1. From Gardner et al. (1974)

Note that no data are shown and that the relation only applies for V_P above about 5000 ft/s (1524 m/s). Consequently, there is no reason to think that the relation should hold for smaller values of V_P. Here is a plot of density and *P*-wave velocity, both measured in Quaternary sediments and from Gardner's relation. The values of V_P less than about 1500 m/s are presumably in unsaturated sediments.

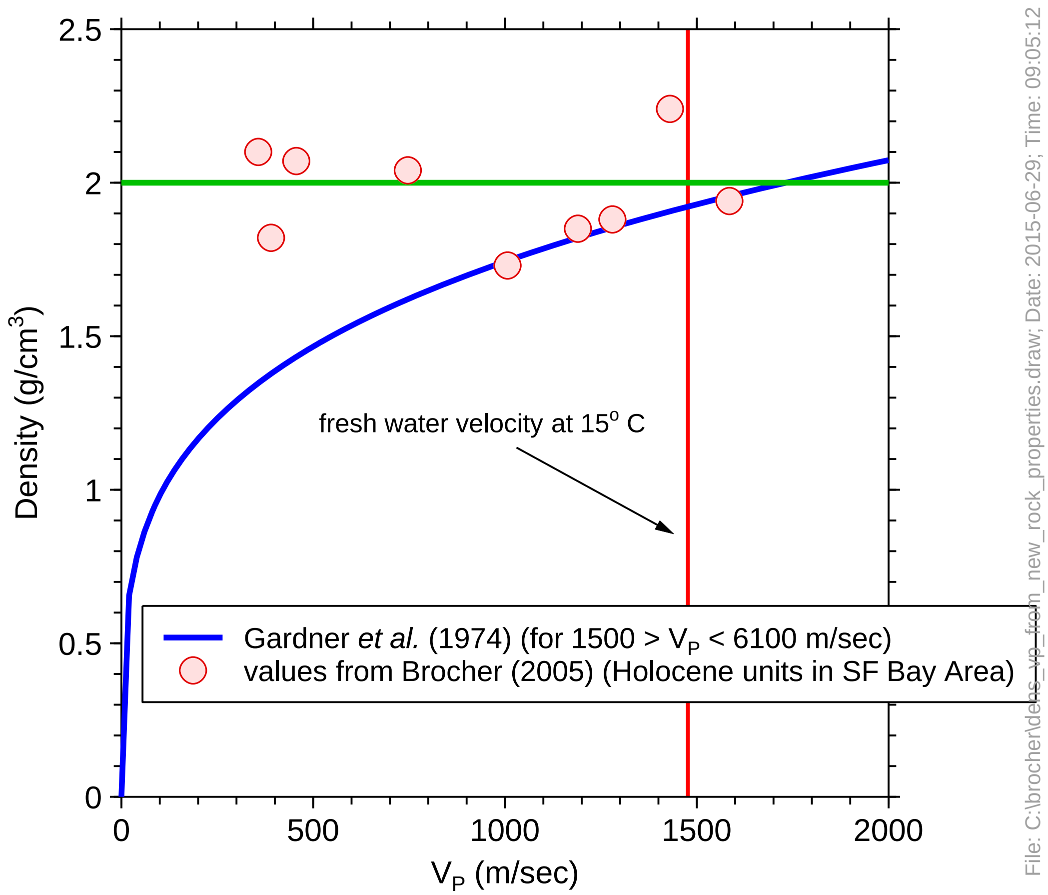

Figure 2. The vertical red line is the velocity of P-waves in fresh water at 15 degrees C (1477 m/s, Press, Table 9-6 in Clark, 1966); the velocity in salt water is above 1500 m/s.

The Gea74 relation appears to be a poor fit for unsaturated near-surface sediments (for which V_P is less than about 1500 m/s). This conclusion, however, is based on a relatively few number of data. A more extensive dataset shown later indicates that the densities for low values of velocity are generally less than the 2.0 g/cm^3 suggested in Figure 2, but even so the Gea74 relation between shear-wave velocity and density is a poor fit to the data for low values of shear-wave velocity. What values of density should be expected? The bulk density ($\bar{\rho}$) of a rock composed of solids and fluid-filled voids is:

$$\bar{\rho} = \rho_S (1-\phi) + \rho_V \phi \qquad (4)$$

where ρ_S , ρ_V , and ϕ are the densities of the solid material, the material filling the voids, and the porosity, respectively. For water-filled voids, $\rho_V = 1$; for air-filled voids $\rho_V = 0$. Here are predicted bulk densities for a range of solid densities and porosities:

Table 1.

Porosity (ϕ)	$\overline{\rho}$ (saturated)	$\overline{\rho}$ (dry)
0	2.65	2.65
0.1	2.49	2.39
0.2	2.32	2.12
0.3	2.16	1.86
0.4	1.99	1.59
0.5	1.83	1.33
0.6	1.66	1.06
0.7	1.50	0.80
0.8	1.33	0.53
0.9	1.17	0.27

where a value of $\rho_S = 2.65$ g/cm^3 was used ($\rho_S = 2.65$ g/cm^3 for quartz and $\rho_S = 2.54 - 2.76$ g/cm^3 for feldspars, according to Lambe and Whitman (1969, Table 3.1).

Here is a plot of the ratio of bulk density for dry and fully-saturated rocks, as a function of porosity.

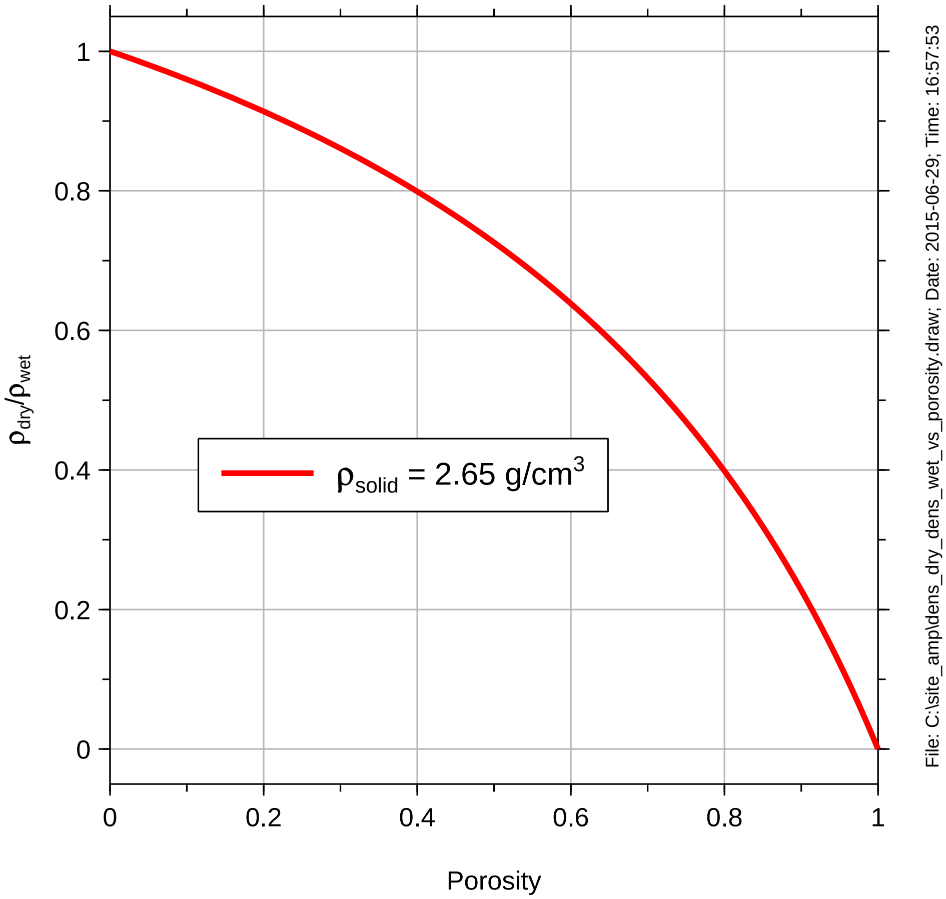

Figure 3.

So the important question is: "what is the porosity"? Porosity depends on material, consolidation, size distribution, and so on. For materials of most relevance, the porosities are generally less than 0.5 (Lambe and Whitman, 1969, Table 3.2).

According to the Wikipedia entry (*http://en.wikipedia.org/wiki/Porosity*):

> **Porosity of soil**
>
> Porosity of surface soil typically decreases as particle size increases. This is due to soil aggregate formation in finer textured surface soils when subject to soil biological processes. Aggregation involves particulate adhesion and higher resistance to compaction. Typical bulk density of sandy soil is between 1.5 and 1.7 g/cm^3. This calculates to a porosity between 0.43 and 0.36. Typical bulk density of clay soil is between 1.1 and 1.3 g/cm^3. This calculates to a porosity between 0.58 and 0.51. This seems counterintuitive because clay soils are termed *heavy*, implying *lower* porosity. Heavy apparently refers to a gravitational moisture content effect in combination with terminology that harkens back to the relative force required to pull a tillage implement through the clayey soil at field moisture content as compared to sand.

> Porosity of subsurface soil is lower than in surface soil due to compaction by gravity. Porosity of 0.20 is considered normal for unsorted gravel size material at depths below the biomantle. Porosity in finer material below the aggregating influence of pedogenesis can be expected to approximate this value.
>
> Soil porosity is complex. Traditional models regard porosity as continuous. This fails to account for anomalous features and produces only approximate results. Furthermore it cannot help model the influence of environmental factors which affect pore geometry. A number of more complex models have been proposed, including fractals, bubble theory, cracking theory, Boolean grain process, packed sphere, and numerous other models.

Using the spreadsheet table above with porosities of 0.2 to 0.35 gives wet and dry densities in good agreement with the values in Figure 2.

Figure 4 is another plot of velocity vs density. The base is a graph I scanned out of a report by Nafe and Drake (I think) that I found on my computer. I've superimposed data from Brocher (2005a) for Quaternary sediments (references indicated in the graph). There is a lot on this graph, because it is a working plot. Note that the Nafe and Drake plot included both *P*- and *S*-wave velocities. Let's look at the *P*-wave velocities first. Based on the graph below and the considerations above, I propose the following model relating density (units of g/cm^3) and *P*-wave velocity (units of km/s):

$V_P < 1.50$ km/s:

> In the previous version, $\rho = 1.93\ \text{g/cm}^3$, but this has been superseded by a new relation for low values of shear-wave velocity. I have not updated the P-wave relation, as the intent of the equations just below is to add curves to the Nafe and Drake plot shown in Figure 4.

$1.50\ \text{km/s} \le V_P < 6.0\ \text{km/s}$:

$$\rho = 1.74 V_P^{0.25} \quad \text{(Gardner et al., 1974)} \qquad \text{(eq. 2, reproduced here)}$$

$6.0\ \text{km/s} \le V_P$:

$$\rho = 1.6612 V_P - 0.4721 V_P^2 + 0.0671 V_P^3 - 0.0043 V_P^4 + 0.000106 V_P^5$$

$$\text{(Brocher, 2005b, eq. 1)} \qquad (5)$$

I chose the Gea74 relation between velocity and density for $1.50\ \text{km/s} \le V_P < 6.0\ \text{km/s}$ because it does a somewhat better job of fitting the Quaternary data than Nafe and Drake (as given by B05b eq. 1) for intermediate values of V_P. In addition, the Gea74 relation seems to be the standard in the exploration geophysics, where it is referred to as "Gardner's Rule" (Sheriff and Geldart, 1995). I chose the break point at

$V_P = 1.5$ km/s because it is between the velocities in fresh and salt water and because it yields a density somewhat below 2.0 g/cm^3 (I think it is better to err slightly on the side of densities that are too small, as that will increase the amplification; in addition, many soils near the surface in dry materials probably have densities somewhat less than 2.0 g/cm^3. Conversely, B05b does a better job of fitting Nafe and Drake for larger values of V_P (as it should).

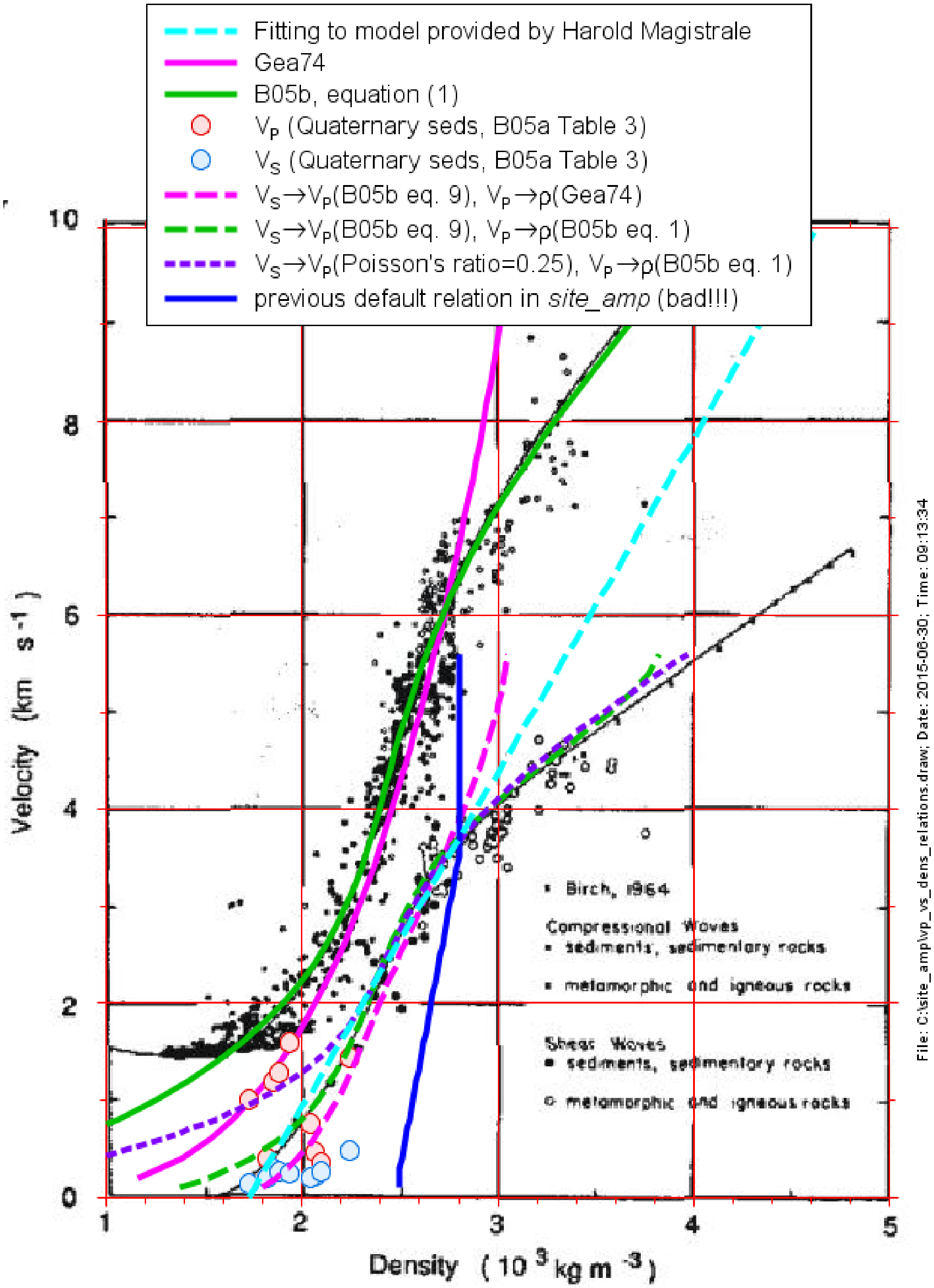

Figure 4. Base figure from Nafe and Drake (I think). B05a = Brocher (2005a); B05b = Brocher (2005b); Gea74 = Gardner et al. (1974).

Relating Density to Shear-Wave Velocity:

At first thought it would seem that the relations given in equations (2) and (5), in combination with equations relating V_S and V_P could be used to relate density and shear-wave velocity. Brocher (2005b, eq. 9) gives the following relation between V_S and V_P (units in km/s):

$$V_P\,(\text{km/s}) = 0.9409 + 2.0947V_S - 0.8206V_S^{\,2} + 0.2683V_S^{\,3} - 0.0251V_S^{\,4} \quad (6)$$

A potential problem is that near-surface data (largely at depths less than 100 m) indicate that the relation should be multivalued, because below the water table V_P is controlled by the water velocity and therefore the Poisson's ratio jumps to a large value. The multivalued relationship is shown in the graph below. The data come from velocity models fit to many borehole measurements (Boore, 2003).

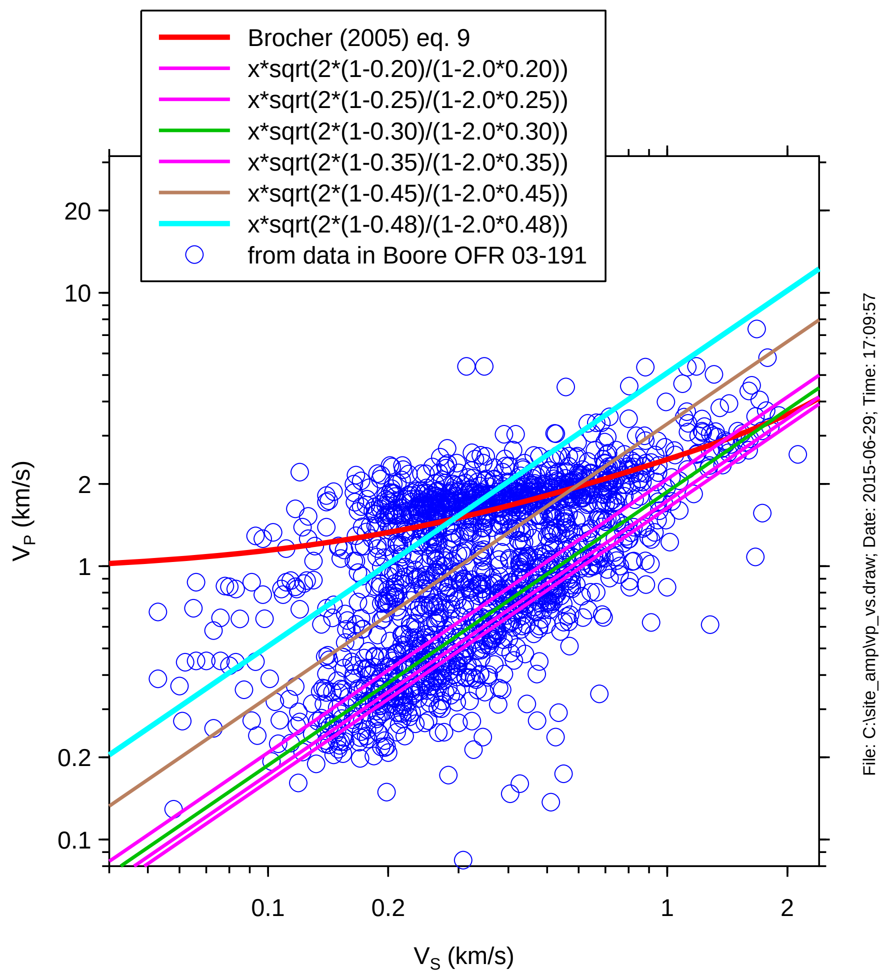

Figure 5.

The straight lines in Figure 5 assume Poisson ratio (σ) values of 0.20, 0.25, 0.30, and 0.35. The green line (for $\sigma = 0.30$) seems like a good representation of the average value.

I will ignore the complication due to the multivalued nature of the relation between V_S and V_P, which may be less important than it would first appear: values of density for soils with relatively low-shear wave velocities may be similar (and close to 2.0 g/cm^3) for both dry and saturated soils. With this assumption, Figure 4 suggests that the following procedure, which is parallel to that proposed above for the $\rho - V_P$ correlation.

In the earlier version of these notes I used $\rho = 1.93\ \text{g/cm}^3$ for $V_S < 0.30\ \text{km/sec}$. This was based on little data. Now I have reviewed data sets for low values of shear-wave velocity from P. Anbazhagan (written commun., 2014 and 2015), Anbazhagan et al. (2015), T. Inazaki (2006), and a number of publications from P. Mayne (in particular, Mayne, 2001, and Mayne et al., 2002). I adjusted the coefficients of a function used by Mayne et al. (1999) such that a reasonable subjective fit to the data was achieved and the function joined the relation in the previous version for $V_S > 0.30\ \text{km/sec}$.

Procedure for relating density (in g/cm^3) to shear-wave velocity (in km/s)

$V_S < 0.30$ km/s:

$$\rho = 1 + \frac{1.53 V_S^{0.85}}{0.35 + 1.889 V_S^{1.7}} \qquad (7)$$

$0.30\ \text{km/s} < V_S < 3.55\ \text{km/s}$:

Use Brocher's (2005b) relation between *S*-wave velocity and *P*-wave velocity (his equation 9, reproduced as equation 6 above), in combination with Gardner et al.'s (1974) relation between *P*-wave velocity and density (equation 2 above). For convenience, I copy the equations here:

$$\rho = 1.74\, V_P^{0.25} \qquad (2)$$

$$V_P(\text{km/s}) = 0.9409 + 2.0947 V_S - 0.8206 V_S^2 + 0.2683 V_S^3 - 0.0251 V_S^4 \qquad (6)$$

$3.55\ \text{km/s} \le V_S$:

Use Brocher's (2005b) relation between *S*-wave velocity and *P*-wave velocity (his equation 9, reproduced as equation 6 above), in combination with Brocher's

(2005b) relation between *P*-wave velocity and density (his equation 1, reproduced as equation 5 above).

$$\rho = 1.6612V_P - 0.4721V_P^{\,2} + 0.0671V_P^{\,3} - 0.0043V_P^{\,4} + 0.000106V_P^{\,5} \tag{5}$$

$$V_P\,(\text{km/s}) = 0.9409 + 2.0947V_S - 0.8206V_S^{\,2} + 0.2683V_S^{\,3} - 0.0251V_S^{\,4} \tag{6}$$

Figures 6a and 6b shows the relation between density and shear-wave velocity obtained from the above procedure is superimposed on a collection of measurements from Mayne (2001) (also in Mayne et al., 2002).

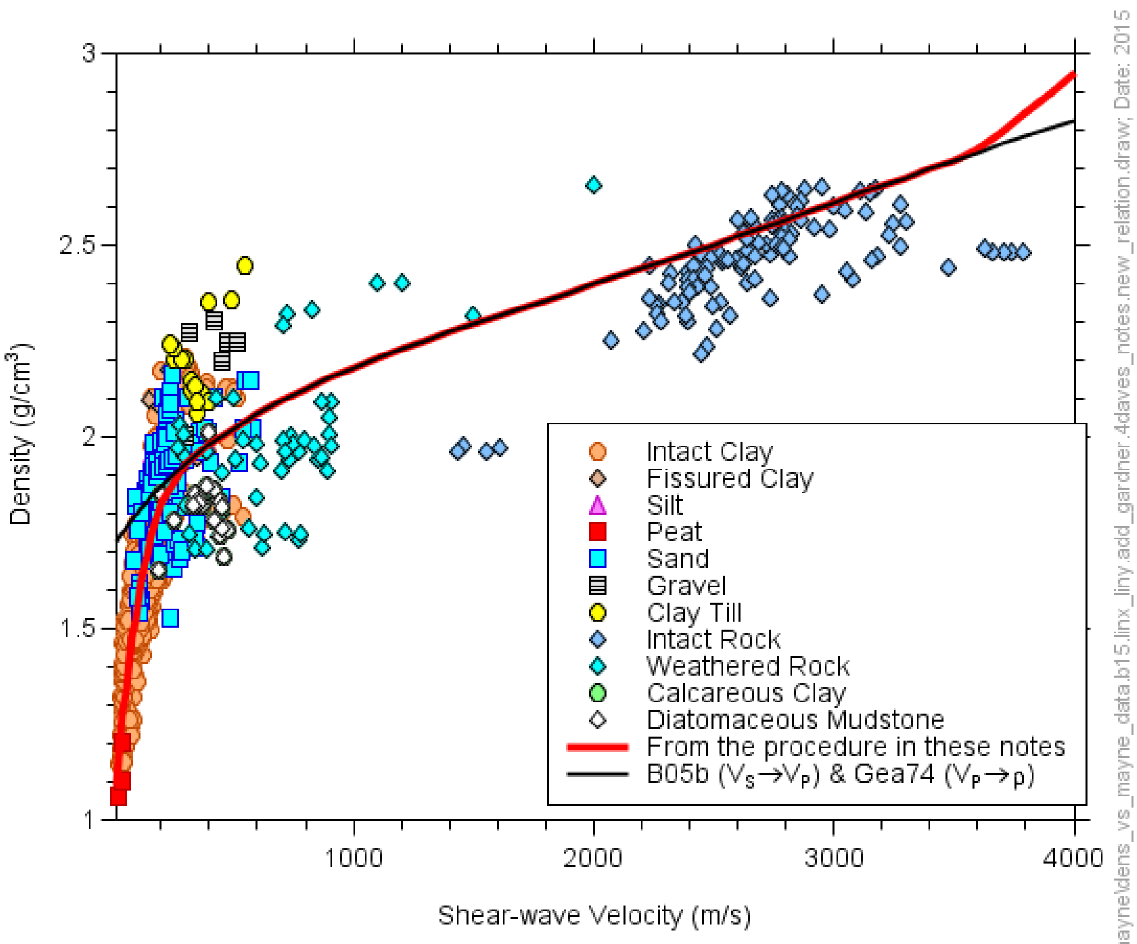

Figure 6a. The relation between density and shear-wave velocity given in these notes and data from Mayne (2001, also in Mayne et al., 2002), using a linear scale for the abscissa. Also shown is the relation using Brocher (2005b) (B05b) to go from V_S to V_P and Gardner et al (1974) (Gea74) to go from V_P to ρ.

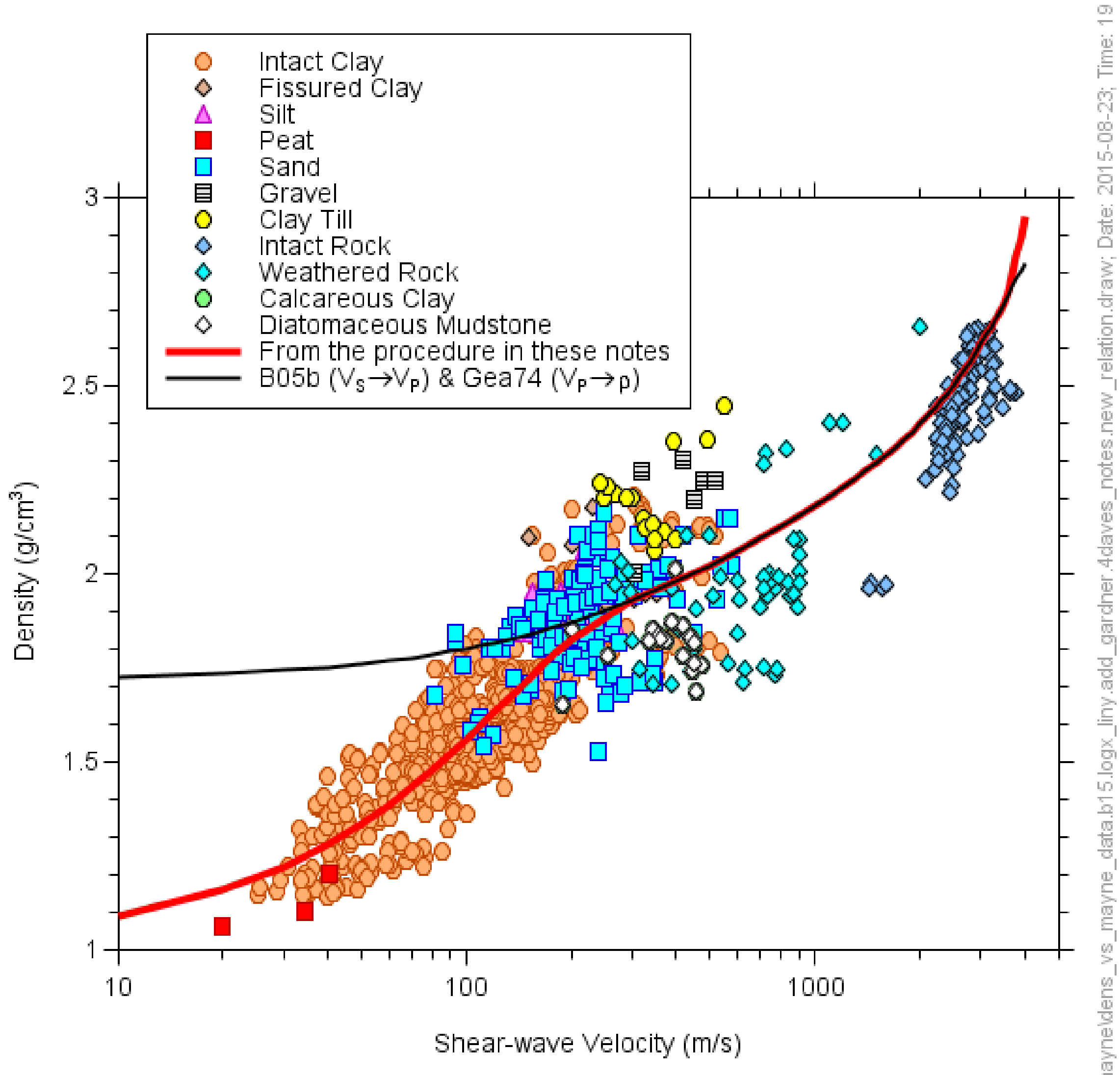

Figure 6b. The relation between density and shear-wave velocity from these notes and data from Mayne (2001, also in Mayne et al., 2002), using a logarithmic scale for the abscissa. Also shown is the relation using Brocher (2005b) (B05b) to go from V_S to V_P and Gardner et al (1974) (Gea74) to go from V_P to ρ.

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{pp. 192--200 contain an excellent discussion of rock densities, including graphs from Birch indicating that soils and alluvia have densities from about 1.65 to 2.2, with peaks in the distribution at 1.7 and 1.9 (the latter being larger); also included is a plot of the Nafe and Drake P-wave vs density data.}

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{contains a chapter on velocity—density relations; the chapter makes heavy use of Castagna et al., 1993---referenced above. Note that most of the figures in the Mavko book are available in a pdf file from
http://pangea.stanford.edu/courses/gp262/Notes/16.avo.pdf.}

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{excellent discussion of porosity and wave speeds in particulate media}.