Relations between corner frequency, source radius, and stress drop

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Comparison of Boore (2003) (actually due to Brune, 1970, 1971) and Allmann and Shearer (2009) (using Eshelby and Madariaga) stress drop—source corner frequency relations

From Boore (2003):

where the constant can be related to the stress drop ($\Delta\sigma$). Following BRUNE (1970, 1971), the corner frequency is given by the following equation:

$$f_0 = 4.9 \times 10^6 \beta_s (\Delta\sigma / M_0)^{1/3} \ , \qquad (4)$$

where f_0 is in Hz, β_s (the shear-wave velocity in the vicinity of the source) in km/s, $\Delta\sigma$ in bars, and M_0 in dyne-cm.

Or

$$\Delta\sigma = M_0 \left(\frac{f_0}{4.9 \times 10^6 \beta_s} \right)^3 \qquad (1)$$

Not mixing units (e.g., β in cm/s, M_0 in dyne-cm, $\Delta\sigma$ in dyne/cm^2, f_0 in s^-1), these are the relations:

From Allmann and Shearer (2009):

individual events. Assuming a circular fault, the stress drop $\Delta\sigma$ can be estimated from the corner frequency f_c of the source spectrum and the seismic moment M_0 using the following relations [*Eshelby*, 1957; *Madariaga*, 1976]:

$$\Delta\sigma = \frac{7}{16}\left(\frac{M_0}{r^3}\right), \ f_c = 0.32\frac{\beta}{r}, \ \rightarrow \ \Delta\sigma = M_0\left(\frac{f_c}{0.42\beta}\right)^3, \quad (3)$$

where r is the source radius and β is the shear wave velocity near the source. We use a constant β of 3.9 km/s and assume the rupture velocity to be 0.9 β. This assumption of a circular fault may not be accurate for all events, especially for

The $f_c - r$ relation is different than that of Brune, and this leads to a different equation relating $\Delta\sigma - f_c$.

$$\Delta\sigma = 13.5 M_0 \left(\frac{f_c}{\beta} \right)^3 \tag{2}$$

Boore (2003):

$$\Delta\sigma = 8.5 M_0 \left(\frac{f_0}{\beta_s} \right)^3 \tag{3}$$

Now considering Boore (2003) only:

More precisely, using the equations in Brune (1970, 1971):

$$\Delta\sigma = 8.47 M_0 \left(\frac{f_0}{\beta_s} \right)^3 \tag{4}$$

This is a 0.3% difference. To get the more precise relation, I should change my basic relation (eq. 4 in Boore, 2003) to

$$f_0 = 4.906 \times 10^6 \beta_s \left(\Delta\sigma / M_0 \right)^{1/3} \tag{5}$$

In fact, this is what I use in rv_td_subs.for. Here is a code snippet:

```
      if (numsource .eq. 1) then

* Single corner frequency:
        stress = stressc*10.0**(dlsdm*(amag-amagc))
        fa = (4.906e+06) * beta * (stress/am0)**(1.0/3.0)
        fb = fa
```

Content added on 14 June 2021: derivation of equations 4 and 5:
I used Brune's equation (36), as corrected in the 1971 errata:

$$\alpha = \frac{2.34\beta_s}{r} \tag{S1}$$

And from Brune's equation (20), α must be related to f_0 by

$$\alpha = 2\pi f_0 \tag{S2}$$

From these two equations, the radius can be written in terms of the corner frequency as

$$r = \left(\frac{2.34}{2\pi}\right) \beta_s \,(1/f_0\,) \tag{S3}$$

Brune's corrected equation (30) is

$$u_d = (\Delta\sigma/\mu) r (16/7\pi) \tag{S4}$$

With the definition of seismic moment

$$M_0 = \mu A u_d = \mu\pi r^2\, u_d \tag{S5}$$

substituting equations (S3) and (S4) into (S5) gives a relation between M_0, f_0, $\Delta\sigma$, and β_s, which can be solved to give equations (4) and (5):

$$\Delta\sigma = 8.4697 M_0 \left(\frac{f_0}{\beta_s}\right)^3 \tag{S6}$$

and

$$f_0 = 0.49058 \beta_s \left(\frac{\Delta\sigma}{M_0}\right)^{\frac{1}{3}} \tag{S7}$$

For all variables in the same units (include 10^7 , as in equation (5), for mixed units as given below equation (1)).
End of content added on 14 June 2021: derivation of equation (5):

Equation for source radius:

$$M_0 = \mu \bar{d} \pi r^2 \tag{6}$$

$$\Delta\sigma = \frac{7\pi}{16}\mu\frac{\bar{d}}{r} \tag{7}$$

Combining:

$$r = \left[\left(\frac{7}{16}\right)\frac{M_0}{\Delta\sigma}\right]^{1/3} \tag{8}$$

In terms of source radius and corner frequency, from equations (4) and (8):

$$r = 0.3724\,\beta_s / f_0 \tag{9}$$

With units of r, M_0, and $\Delta\sigma$ of km, dyne/cm, and bars this becomes:

$$r = 7.59\times10^{-8}\left(M_0/\Delta\sigma\right)^{1/3} \tag{10}$$

and

$$\Delta\sigma = 4.372\times10^{-22}\,M_0/r^3 \tag{11}$$

This gives the following table:

M	Ds	r
3	50	0.15
4	50	0.46
5	50	1.46
6	50	4.61
7	50	14.59
8	50	46.12
3	100	0.12
4	100	0.37
5	100	1.16
6	100	3.66
7	100	11.58
8	100	36.61
3	200	0.09
4	200	0.29
5	200	0.92
6	200	2.91
7	200	9.19

8	200	29.06

References

Allmann, B. P. and P. M. Shearer (2009). Global variations of stress drop for moderate to large earthquakes, *J. Geophys. Res.* **114**, B01310, doi:10.1029/2008JB005821, 22 pp.

Boore, D.M. (2003). Prediction of ground motion using the stochastic method, *Pure and Applied Geophysics* **160**, 635–676. Available from the online publications link on http:// www.daveboore.com.

Brune, J. N. (1970). Tectonic stress and the spectra of seismic shear waves from earthquakes, *J. Geophys. Res.* **75**, 4997–5009.

Brune, J. N. (1971). Correction, *J. Geophys. Res.* **76**, 5002.