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Choosing the Lowest Usable Frequency for Response Spectra from Filtered Data

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Introduction

The question at hand is what is the lowest frequency for which response spectra from low-cut (high-pass) filtered data are relatively unaffected by the filter? This is most conveniently expressed by specifying the ratio $\eta = f_u/f_c$, where f_u is the lowest usable frequency and f_c is the frequency of the low-cut filter. With this notation, response spectra can be used for oscillator frequencies (f_o) such that

$$f_o \geq \eta f_c.$$

Or in terms of the more familiar oscillator period (T_o),

$$T_o \leq 1/(\eta f_c).$$

I find that η based on the filter frequency-domain response function may be too small, in that response spectrum at $f_o = f_u$ can be affected by the filter more strongly than is the Fourier amplitude spectrum. In other words, the usable band for the response spectrum may start at smaller periods than given by the analysis based on the filter response. The disagreement increases with filter order and is more important for longer than shorter period filters. The results suggest that on average $\eta = 2$ results in a reasonably close approximation to the -1/2db value of the response spectra of the unfiltered data, independent of filter order for filter frequencies of 0.05 and 0.10 Hz; this value of η is conservative for filter corners of 0.2 and 0.4 Hz (these results are specific to the one record studied here).

Analysis in Terms of Filter Response in the Frequency Domain

One way of deriving η is from the equation for the filter response, finding f_u/f_c such that the filter response equals η. I do that now. The equation for 1-pass of a Butterworth filter is:

$$F_{1p} = \sqrt{(f/f_c)^{2n}/[1 + (f/f_c)^{2n}]}. \quad (1)$$

For USGS processing using a time-domain acausal (zero phase shift) filter, the record is passed through the filter in the forward and reverse direction (2-pass), whereas acausal filtering in the frequency domain is free to choose any filter shape. Pacific Engineering and Analysis uses the 1-pass shape for their frequency-domain acausal filtering. The USGS acausal response is:

$$F_{2p} = F_{1p}^2 = (f/f_c)^{2n}/[1 + (f/f_c)^{2n}]. \tag{2}$$

If we want the response to be η of the full response (unity), then we can solve the above equations for the required ratio of f to f_c. This will be a function of the filter order n. But before doing that, note that the filter order used in BAP and my software is $nroll$. The parameter $nroll$ was introduced by Converse and Brady (1992), and is related to the order (n) of a causal Butterworth filter by the following equation:

$$n = 2 * nroll.$$

Solving the equations for f/f_c gives:

For causal (1-pass):

$$f_u/f_c = (\frac{\eta^2}{1-\eta^2})^{1/2n}, \tag{3}$$

For acausal (2-pass):

$$f_u/f_c = (\frac{\eta}{1-\eta})^{1/2n}, \tag{4}$$

Now we want to know what value of η corresponds to being a certain db down from the maximum response of unity. This can be obtained using the relation:

$$db = 20 \log \eta$$

so that

$$\eta = 10^{db/20}$$

So for db = -1/4, -1/2, -1 we have η = 0.97, 0.94, and 0.89. Note that increasing the db by a factor of 2 corresponds to squaring η. But from equations (3) and (4), we see that the curves of f_u/f_c vs n for a 1-pass filter for a specified db below the maximum response will be the same as the 2-pass filter with 2 times the 1-pass db factor. Figure 1 shows the relation. Table 1 is a summary for a number of low-cut filters used in the PEER NGA database:

Do Response Spectra Agree with the Above Analysis?

The above information is based on the Fourier amplitude spectrum of the filter response. What is most important, of course, is whether the longest usable period for response spectra can be chosen using the same analysis. Response spectra are a nonlinear operation, unlike Fourier spectra; a response spectrum at a given oscillator period can be influenced by periods of ground motion far from the oscillator period. This makes it more difficult to do the same kind of general analysis used above. We are looking for the oscillator period range that is relatively little affected by the filter used in processing the time series (actually, I am only considering the long-period end of the "range"). The way I have assessed this is to compare response spectra for a single record, filtered using a number of values of filter corner and filter order. The "comparison-of-merit" is the comparison between the response spectrum of the filtered record with the response spectrum of the unfiltered record. In particular, what is the ratio of the response spectra of the filtered and unfiltered records at the period for which the analysis of the filter response predicts that the ratio should be a given factor? In the results shown here I have used a factor of 0.944, which corresponds to the -1/2 db value. I have considered only one time series (the recording of the 1989 Loma Prieta earthquake at the Anderson Dam downstream station). Why this particular record? It is the one used by Converse and Brady to illustrate various record processing procedures, and it was used in a recent paper by Stephens and Boore (2004), so it was convenient for me to use that record. The record has a significant amount of long-period noise (Figure 2), and so it is a good candidate for processing by low-cut filtering.

The Fourier amplitude spectra for the unfiltered record and the record filtered using the same value of f_c (0.05 Hz) and three values of n are shown in Figure 3. (The original record was padded with zeros before the spectrum was computed and before filtering). Except for the very lowest frequencies where the filter response is down by about four orders of magnitude (and thus probably influenced by finite word size in the computer), the spectra of the filtered record behave as expected from equation (2). As predicted by the equation, the spectra for the three values of n cross when $f = f_c$. Another check is given by multiplying the unfiltered Foruier spectrum by equation (2). As shown in the figure, this gives values indistinguishable from the spectra of the filtered data (recall the the spectra in Figure were obtained from time-domain filtering of the unfiltered records, followed by Fourier transformation using a FFT subroutine).

Figures 4 through 6 show graphs of response spectra for four filter corners and one filter order per plot. I use displacement response spectra to more clearly show the detail comparisons at long periods, where the displacement response spectrum (SD) has much smaller variation in the ordinate than does the pseudo acceleration response spectrum (PSA). But because the two types of spectra are related by

$$PSA = (2\pi/T_o)^2 SD$$

the conclusions drawn from the SD graphs will be the same as those from the PSA graphs. As a quick way of seeing the frequency at which the response spectra from filtered records equals the spectrum of the unfiltered record reduced by -1/2db, I show in the plots the spectrum from the unfiltered record reduced by a factor of 0.944. Another comparison are plots of the response spectra for various filter orders and a single filter corner, as shown in Figures 7 through 10 (the spectra are the same as in Figures 4 through 6, just ordered differently). Figures 4 through 10 show that for the two lowest filter corners (0.05 and 0.10 Hz), the response spectra are lower than the spectrum of the unfiltered data by factors larger than expected based on the filter response. The disagreement seems to be larger for 0.10 Hz than 0.05 Hz, although in a reverse of this trend, there seems little disagreement for the filter corners of 0.2 and 0.4 Hz. The disagreement also seems to be larger for higher order filters than lower order filters. In an atttempt to derive an empirical correction to the values of η given in Table 1, I estimated the additional factor by which η would have to be multiplied in order for the filtered and unfiltered spectra to be within 1/2db of one another. The original and modified values of η are given in Table 2. From this table a constant value of $\eta = 2$ is suggested as a useful "rule-of-thumb". This factor would be conservative if applied to the higher frequency filters.

Discussion

Because the illustration here is for a single time series, and because I do not have a subroutine to do the filtering in the frequency domain (easily done, but I do not have time), the conclusions may not be general. But I suspect that overall the conclusion that the values of η given by the analysis based on the filter response in the frequency domain are somewhat unconservative will hold in general, particularly when the corner frequency of the filter is relatively low. By "unconservative" I mean that the actual usable band of response spectra from filtered records may start at smaller values of oscillator period than given by the frequency-domain filter analysis. Based on the results shown here, increasing the value of η given in Table 1 to a constant value of 2 will lead to ratios of response spectra of filtered and unfiltered data as large or larger than the factor of 0.94 (-1/2db) chosen as the limit of the usable bandwidth.

Acknowledgments

Thanks to Walt Silva for finding a few typos and for suggesting the "1-pass", "2-pass" terminology to differentiate between frequency-domain and time-domain acausal Butterworth filters.

References

Converse, A.M. and A.G. Brady (1992). BAP — Basic strong-motion accelerogram processing software; Version 1.0, *U.S. Geol. Surv. Open-File Rept. 92-296A*, 174p.

Stephens, C. D. and D. M. Boore (2004). ANSS/NSMP strong-motion record processing and procedures, *Proc. Workshop on Strong-Motion Record Processing*, Consortium of Organizations for Strong-Motion Observation Systems (COSMOS), May 26–27, 2004, Richmond, California.

Table 1

Type	n-pass	nroll	norder	$\eta(= f_u/f_c)$
causal	1	2	4	1.3
acausal	2	1	2	2.0
acausal	2	2	4	1.4
acausal	2	4	8	1.2

Table 2. η factor needed to come within -1/2db of the response spectrum of the unfiltered record.

Type	n-pass	f_c	nroll	norder	η	extra factor	η (modified)
acausal	2	0.05	1	2	2.0	1.1	2.2
acausal	2	0.05	2	4	1.4	1.3	1.8
acausal	2	0.05	4	8	1.2	1.5	1.8
acausal	2	0.10	1	2	2.0	1.3	2.6
acausal	2	0.10	2	4	1.4	1.5	2.1
acausal	2	0.10	4	8	1.2	1.6	1.9
acausal	2	0.20	4	8	1.2	1.3	1.6

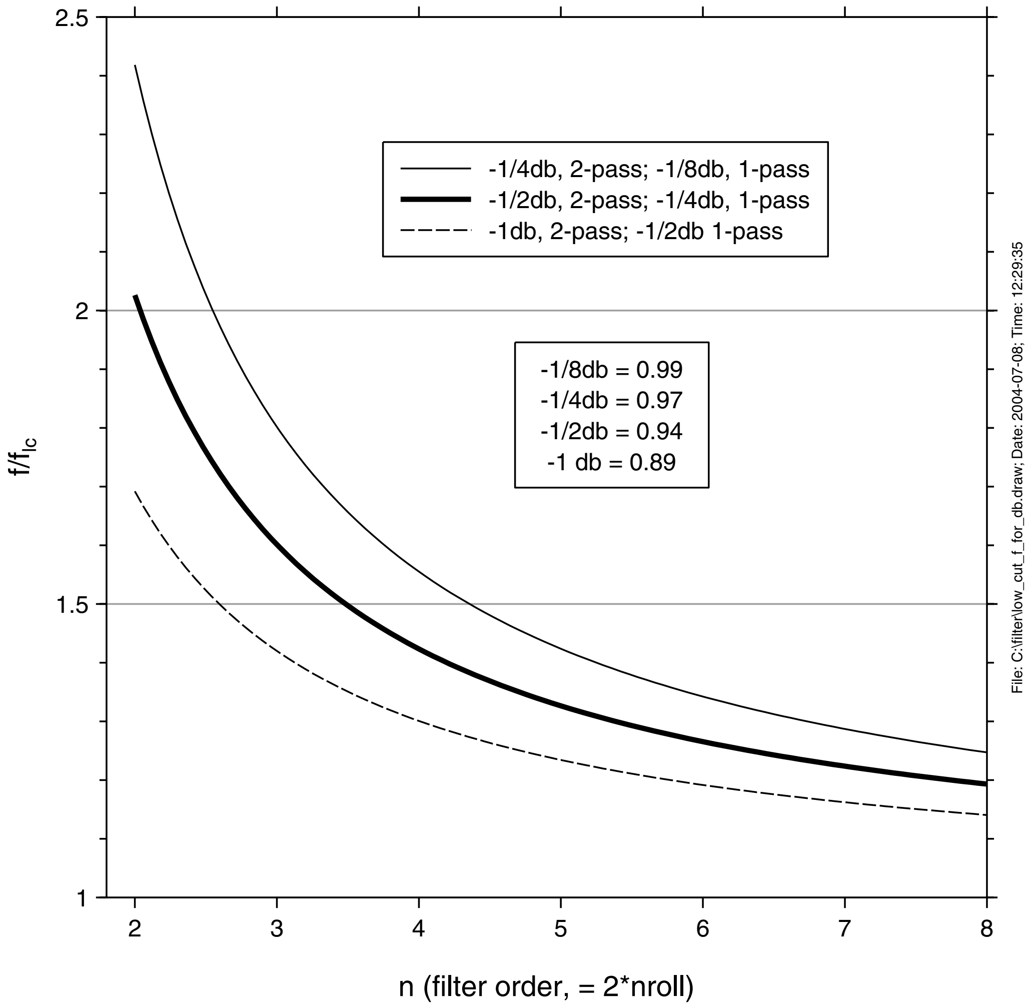

Figure 1. Ratio of frequency to corner frequency in order to be a certain fraction of the peak filter response, as a function of filter order n.

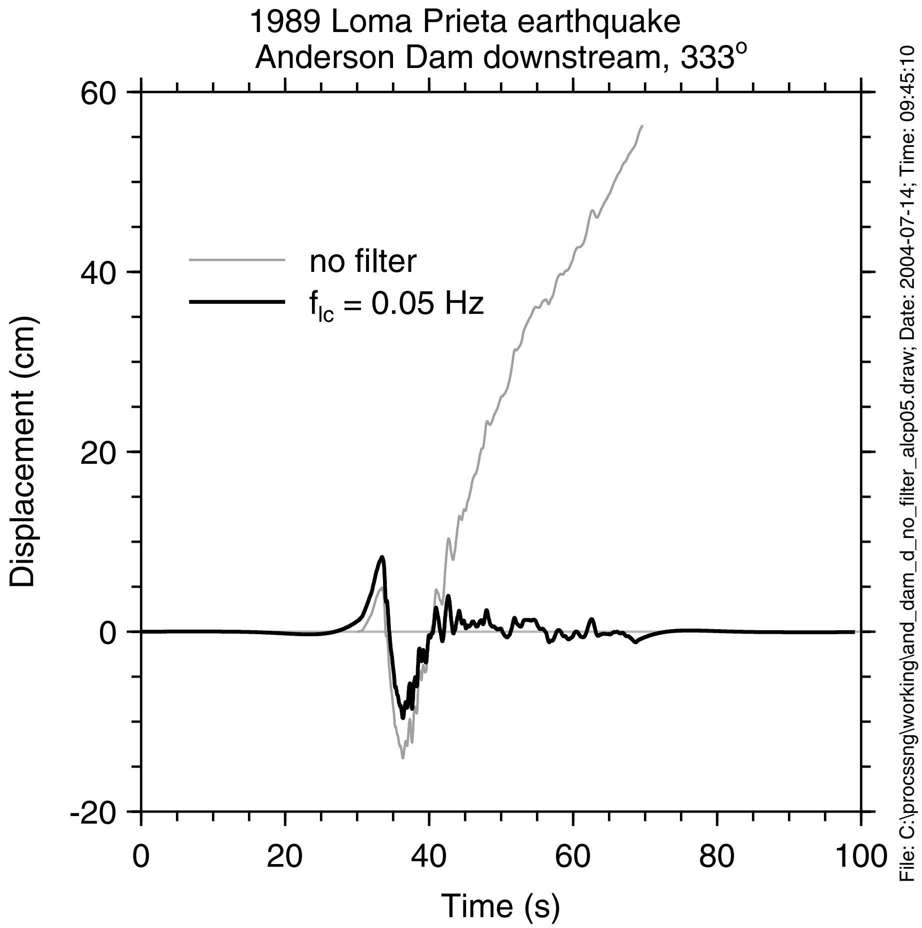

Figure 2. Displacement derived from double integration of the unfiltered and the filtered acceleration time series recorded at Anderson Dam downstream during the 1989 Loma Prieta earthquake. The overall mean of the unfiltered record was removed before integration. The filtering was done using a 2-pass (acausal) 2nd order filter ($n = 2$ in equation (2)) with a filter corner of 0.05 Hz.

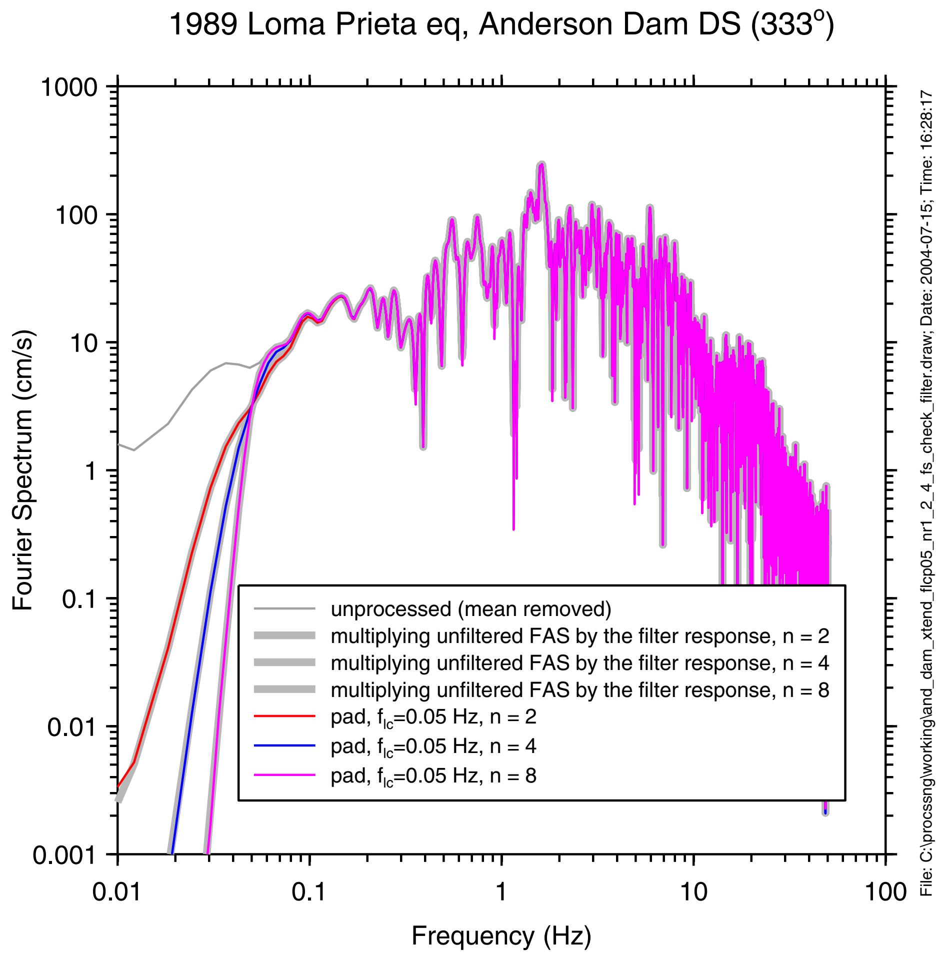

Figure 3. Fourier amplitude spectra of acceleration time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without filtering, demonstrating that the filter is working properly.

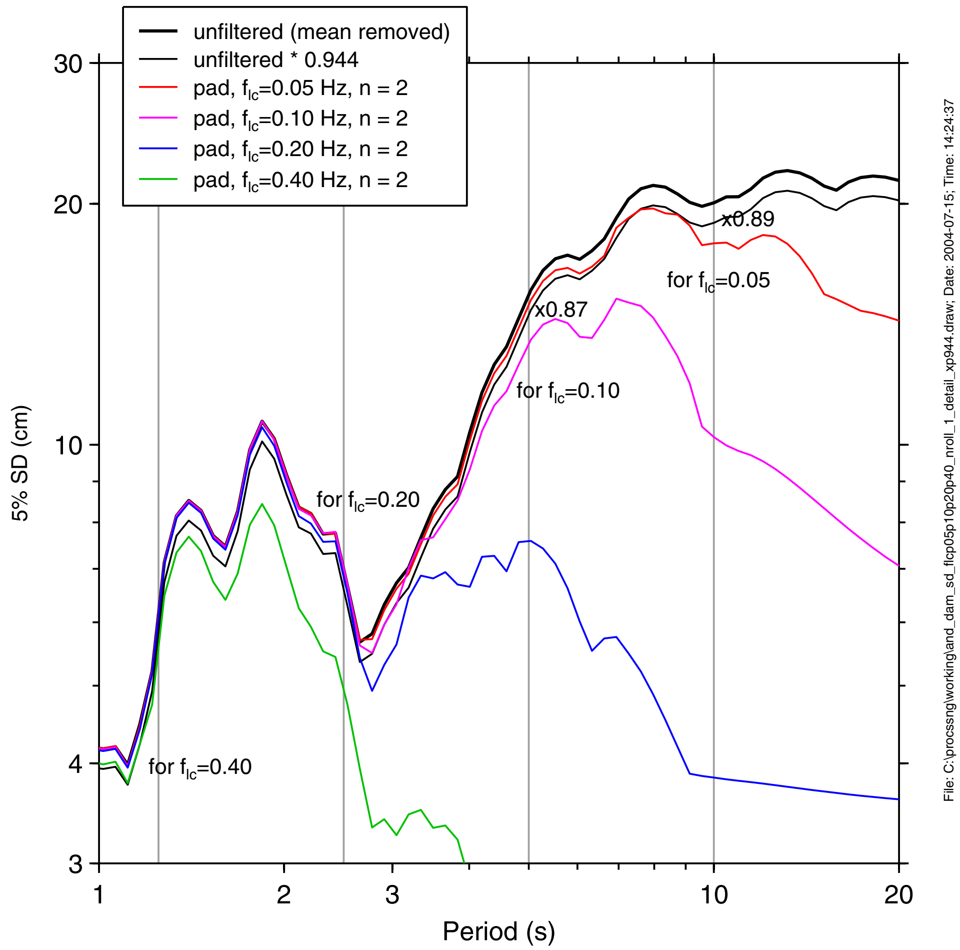

Figure 4. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The acausal filtering is for a series of corner frequencies and a single value of filter order ($n = 2$). The vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with “x” indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

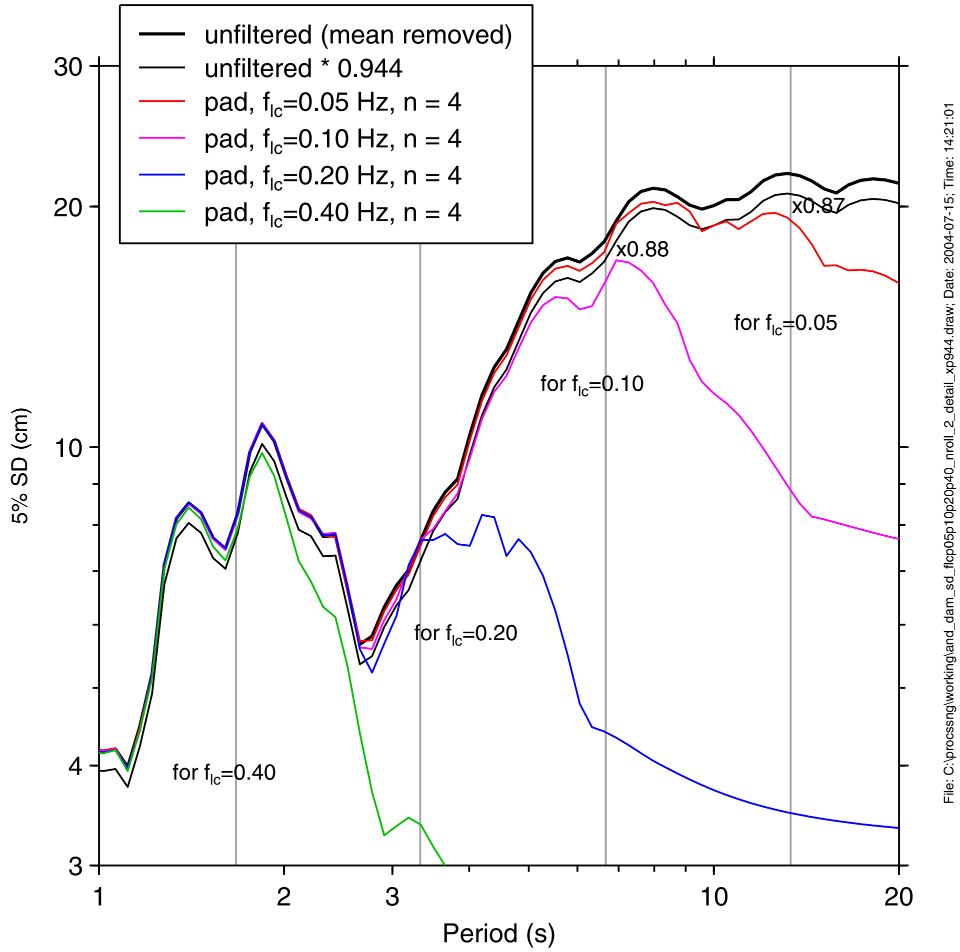

Figure 5. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The acausal filtering is for a series of corner frequencies and a single value of filter order ($n = 4$). The vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

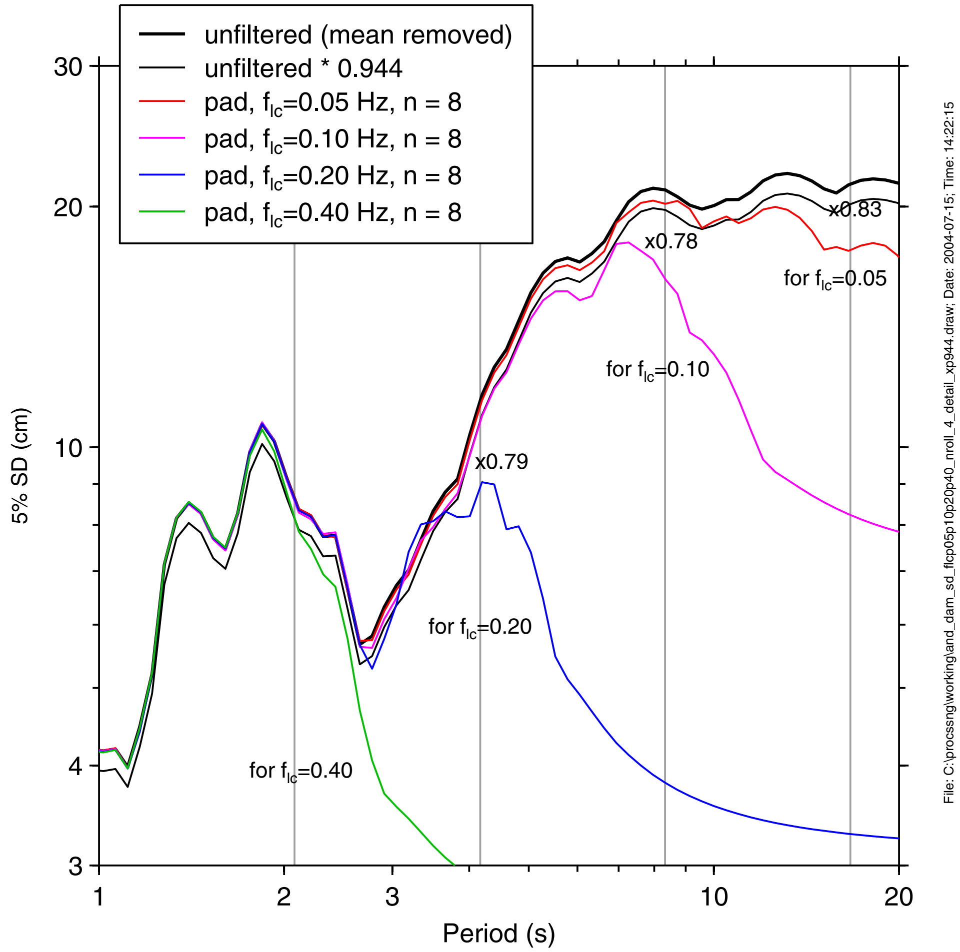

Figure 6. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The acausal filtering is for a series of corner frequencies and a single value of filter order ($n = 8$). The vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

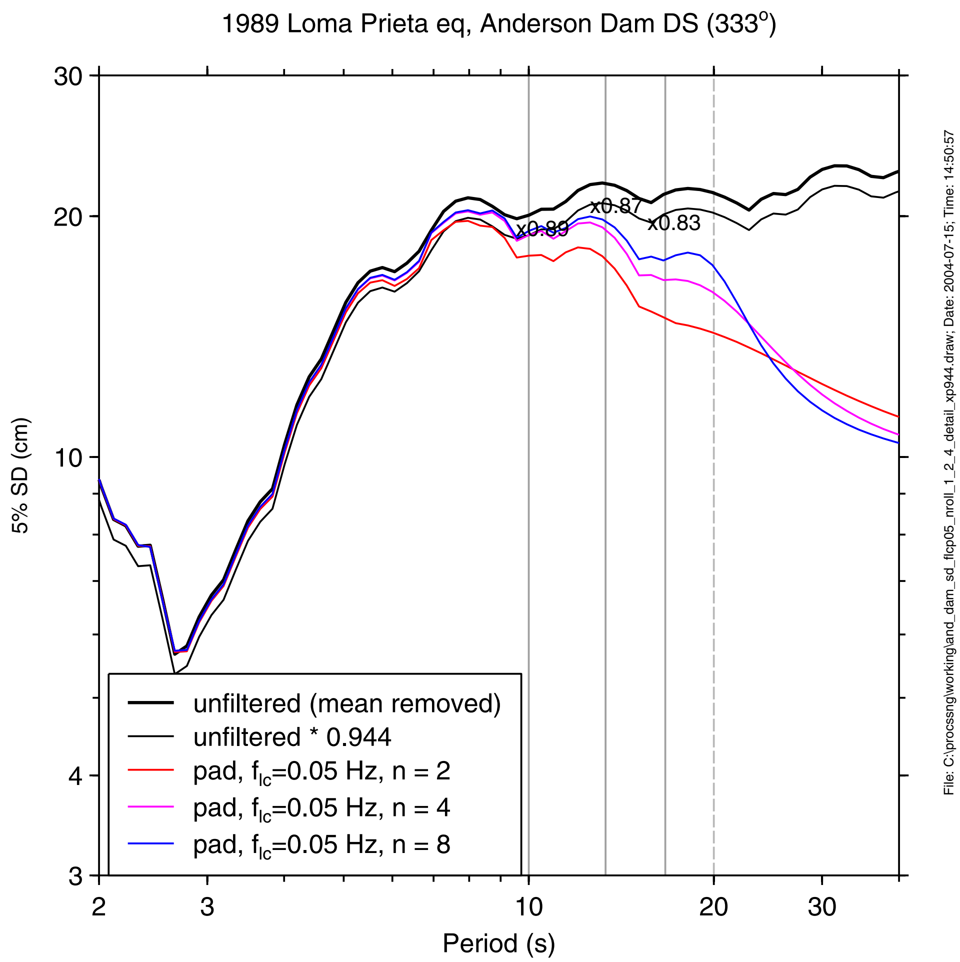

Figure 7. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The filtering is for a series of filter orders and a single value of corner frequency. The dashed vertical line indicates the filter corner. The solid vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

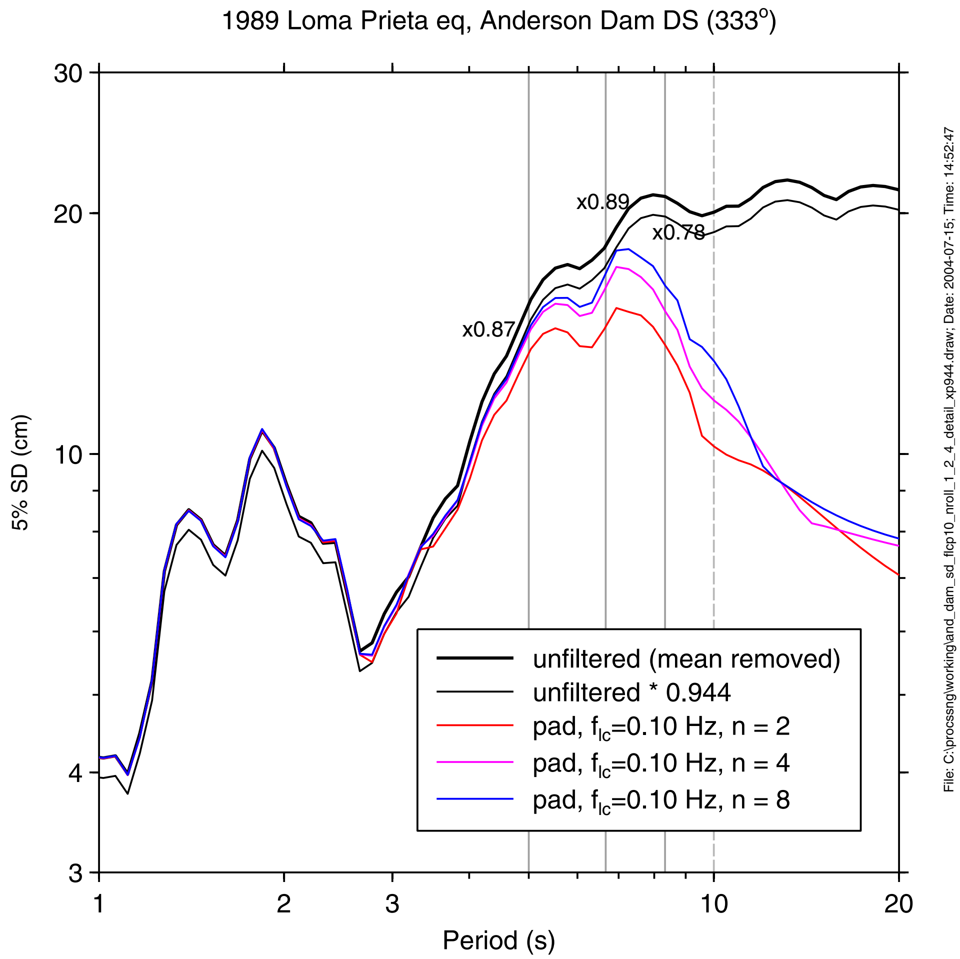

Figure 8. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The filtering is for a series of filter orders and a single value of corner frequency. The dashed vertical line indicates the filter corner. The solid vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

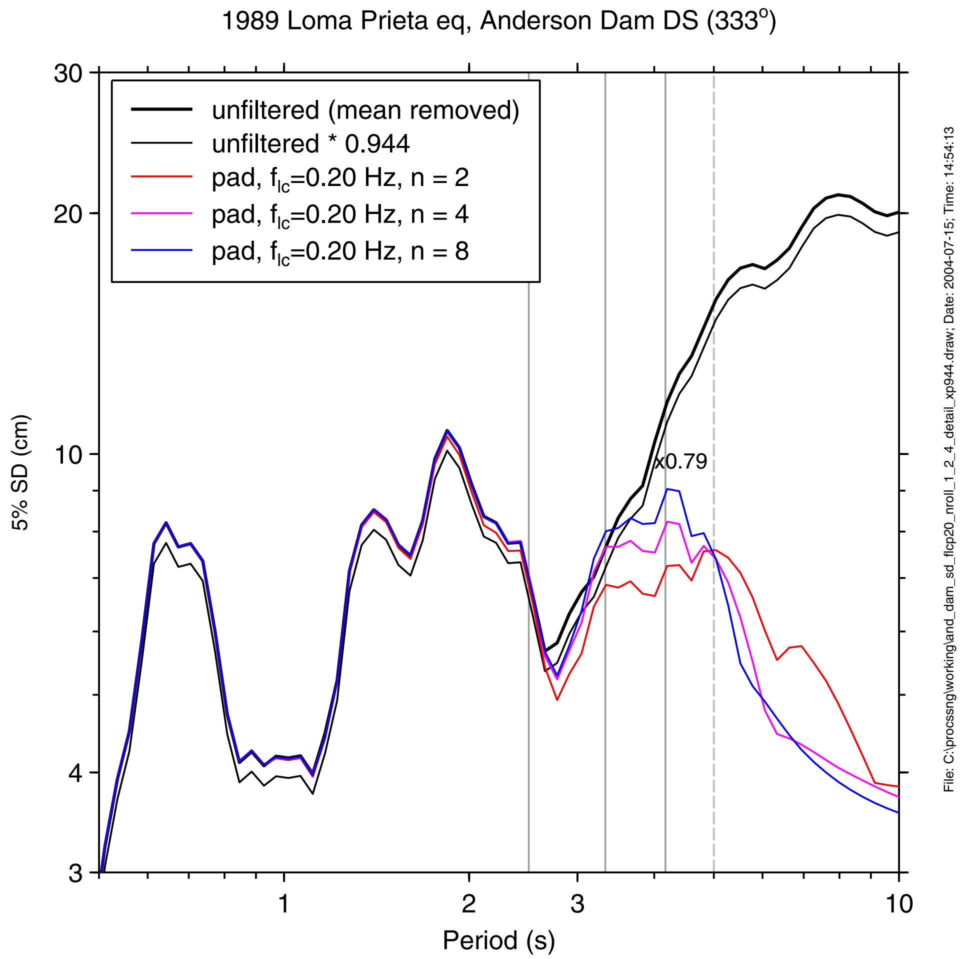

Figure 9. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The filtering is for a series of filter orders and a single value of corner frequency. The dashed vertical line indicates the filter corner. The solid vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).

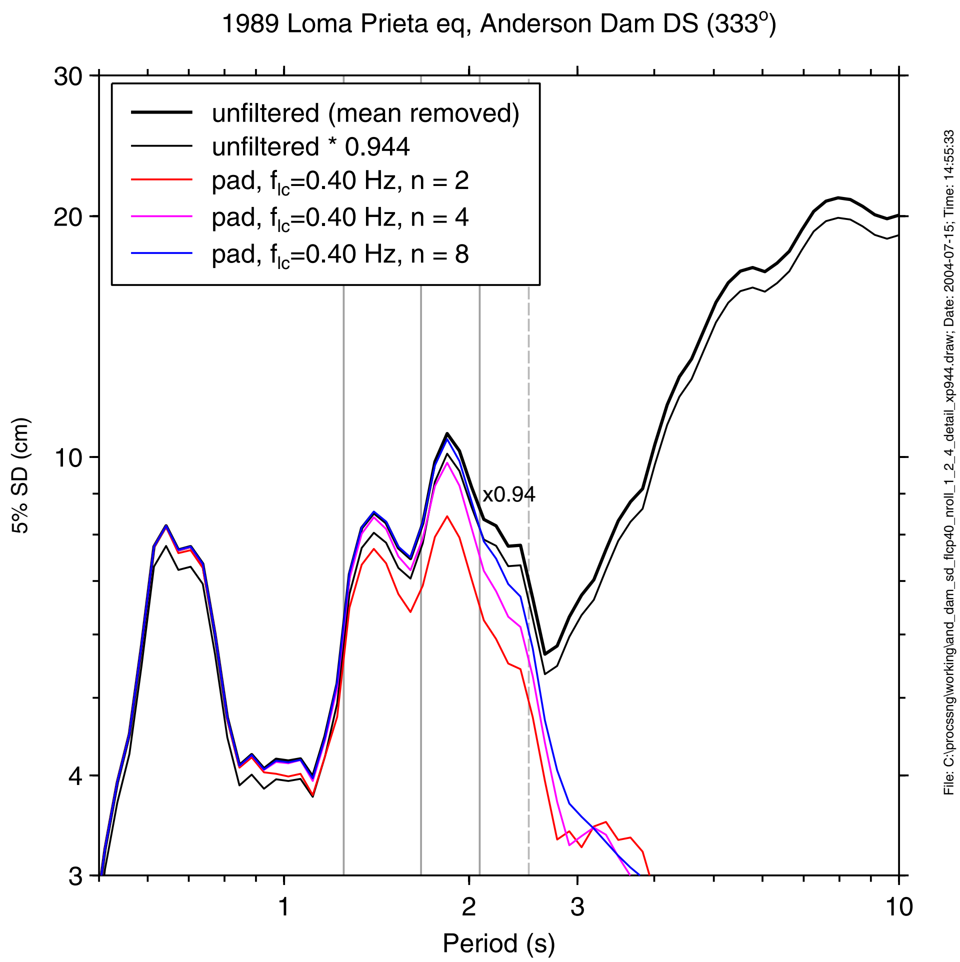

Figure 10. Response spectra from time series at Anderson Dam downstream, 1989 Loma Prieta earthquake, with and without acausal, time-domain (2-pass) filtering. The unfiltered spectrum is shown in two versions: as is (thick line) and multiplied by 0.944 to better compare the filtered response with expectations based on the filter frequency-response analysis (thin line). The filtering is for a series of filter orders and a single value of corner frequency. The dashed vertical line indicates the filter corner. The solid vertical lines denote periods for which the filter response is down by about -1/2db (a factor of 0.94). The text starting with "x" indicates the actual ratio between response spectra for the unfiltered and the filtered acceleration time series (placed at the point appropriate for the -1/2db period for the particular filtered time series).