Notes by D. Boore on July 25, 2001

`rampdis.tex`

Consider the response of an oscillator to finite ramp displacement of final offset D reached in time t_R. This corresponds to the following acceleration forcing function:

$$u_g = (D/t_R)\delta(t) - (D/t_R)\delta(t - t_R).$$

From eq. B.55 in Kramer, the oscillator response is:

$$u_{osc} = (D/t_R)(1/\omega_d)[\exp(-\xi\omega_0 t)\sin(\omega_d t)H(t) - \exp(-\xi\omega_0(t-t_R))\sin(\omega_d(t-t_R))H(t-t_R)]$$

where $H(t)$ is the Heaviside function, ξ is the fractional damping, $\omega_0 = 2\pi/T_{osc}$, and $\omega_d = \omega_0\sqrt{(1-\xi^2)}$. The task is to find the maximum over time of u_{osc}, as a function of T_{osc} for a given damping (call that RD). What is of particular interest is the value of T_{osc} for which RD approaches D. It might help to write things in a normalized form:

$$u_{osc}/D = \frac{\eta}{2\pi\sqrt{(1-\xi^2)}}[\exp(-2\pi\xi\epsilon/\eta)\sin(2\pi\sqrt{(1-\xi^2)}\epsilon/\eta)H(\epsilon)$$

$$-\exp(-2\pi\xi(\epsilon-1)/\eta)\sin(2\pi\sqrt{(1-\xi^2)}(\epsilon-1)/\eta)H(\epsilon-1)],$$

where $\eta = T_{osc}/t_R$ and $\epsilon = t/t_R$. Now RD is defined as the maximum of the absolute value of u_{osc}/D, sweeping over all times ϵ, and what is wanted is a plot of RD vs η. Does RD approach unity for η near unity or for much larger η (as found for some of the Chi-Chi response spectra, such as TCU084, TCU089)?

It is, of course, possible to take analytical derivatives of the oscillator response and set those to zero, but I balk at the algebra. Probably faster is to program the function in Fortran or Matlab and to find the necessary quantities.