Bulletin of the Seismological Society of America, Vol. 74, No. 1, pp. 315–324, February 1984

USE OF SEISMOSCOPE RECORDS TO DETERMINE M_L AND PEAK VELOCITIES

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ABSTRACT

More information about ground motion can be extracted from seismoscope records than a single point on a response spectrum. To demonstrate this, the relation between seismoscope response and Wood-Anderson instrument output and peak horizontal ground velocity has been studied by simulating the various responses for a range of distances and magnitudes. The simulations show that the relation used by Jennings and Kanamori (1979) to convert from peak seismoscope readings to the peak response of a Wood-Anderson instrument has a distance- and magnitude-dependent systematic error. The error is negligible, however, for modern seismoscopes at distances of a few tens of kilometers. At several hundred kilometers, the relation underestimates the Wood-Anderson response by as much as a factor of two. The spread in Jennings and Kanamori's estimate of M_L for the 1906 San Francisco earthquake, recorded on seismoscopes having relatively low natural frequencies (0.26 and 0.5 Hz), is reduced by the results in this paper—the upper value, from a seismoscope in Carson City, Nevada, at 290 km from the fault, going from $M_L = 7.2$ to $M_L = 7.0$ and the lower value, from Yountville, California ($R \approx 60$ km), going from about 6.3 to 6.4. About 0.3 units of the remaining spread may be due to local geologic site conditions. If the 0.3 units is distributed equally beween the Yountville and Carson City recordings, the estimates of M_L for the San Francisco earthquake then become 6.5 and 6.8, somewhat lower than Jennings and Kanamori's final estimates of $6\frac{3}{4}$ to 7. Although the error in using the relation of Jennings and Kanamori to estimate Wood-Anderson response was at most a factor of 1.6 for the 1906 earthquake, the error can be substantially larger for smaller earthquakes recorded on similar low frequency seismoscopes.

The relation between Wood-Anderson and seismoscope response used by Jennings and Kanamori can be combined with an empirical relation between peak horizontal velocity and Wood-Anderson response to predict peak velocity from seismocope recordings. The simulations show that this relation ($v_{max} = 8.1A_{wa}$, where v_{max} is the peak horizontal velocity in centimeters/second and A_{wa} is one-half the range of the Wood-Anderson motion in meters) forms a lower bound for estimates of peak velocity from seismoscope recordings. The relation is good for stations within about 100 km of earthquakes with moment magnitudes of about 4.5 to 6.5, and it underestimates peak velocity by factors up to 2 or 3 for larger earthquakes at distances within 100 km. An application of the simulation method to the 1976 Guatemala earthquake (moment magnitude = 7.6) results in 37 cm/sec as a lower bound to v_{max}, with 66 cm/sec as a more likely value, from the seismocope recording in Guatemala City (approximately 25 km from the Motagua fault).

INTRODUCTION

Seismoscopes are simple, lightly-damped mechanical oscillators whose motion relative to the ground is recorded as a hodograph rather than a time series. They are inexpensive and rugged instruments. The modern seismoscopes have natural periods close to 0.77 sec and dampings near 0.1; these values were chosen with engineering applications in mind, for with them the seismoscope record provides a point on a response spectral curve that is especially relevant to engineering design.

(A typical eight-story building has a natural period close to 0.8 sec and under strong shaking, a damping near 0.1.) For whatever reason, seismoscope records have been generally neglected, at least by seismologists, even though many records are available and would seem to be a valuable supplement to the information on ground shaking provided by accelerographs, especially in countries and regions where budgets are limited and access is difficult. Although a few painstaking studies have reconstructed accelerograms from seismoscope data (Trifunac and Hudson, 1970; Scott, 1973), widespread use of seismoscopes would benefit from a simpler way of extracting ground motion information. In the only effort of this kind known to me, Jennings and Kanamori (1979) have recently extended the usefulness of seismoscope recordings by devising a clever method for determining Richter local magnitudes from the peak response of seismoscopes. In a classic piece of seismological sleuthing, they applied their method to a determination of M_L for the 1906 San Francisco earthquake. This required the location and reconstruction of several of the seismoscopes that recorded the earthquake. (The natural frequencies and dampings for these instruments, given in Table 1, differ from those of modern seismoscopes.) A critical need in their method is a theoretical relation between the peak response of two oscillators subjected to the same ground shaking; such a relation allows the prediction of the peak amplitude of a Wood-Anderson seismograph, and thus the Richter local magnitude, M_L, from the measured seismoscope response. Jennings and Kanamori used

$$\frac{A_{wa}}{A_{sc}} = \frac{V_{wa}}{V_{sc}} \sqrt{\left(\frac{T_{wa}}{T_{sc}}\right)^3 \frac{\zeta_{sc}}{\zeta_{wa}}} \tag{1}$$

TABLE 1

INSTRUMENT CONSTANTS

Instrument	Natural Period (sec)	Damping
Wood-Anderson	0.80	0.80
Modern seismoscope	0.77	0.10
Duplex pendulum	3.8	0.25
Carson City, Nevada (1906)		
Simple pendulum	2.0	0.02
Yountville, California (1906)		0.10

where V, T, and ζ are the static magnification, natural period, and damping, respectively. The subscript wa refers to the Wood-Anderson instrument and sc to the seismoscope. Two basic assumptions have been made in deriving equation (1). The first is that the ratio of peak responses of two oscillators is equal to the ratio of the rms of the oscillator outputs. Simple random vibration theory predicts that the peak (A_{max}) and rms responses (A_{rms}) of a stochastic time series with N extrema are related by

$$A_{max} = A_{rms}[2 \ln(N)]^{1/2} \tag{2}$$

(Cartwright and Longuet-Higgins, 1956; Boore, 1983), and therefore the ratio of rms responses of two oscillators should be similar to the ratio of peak responses if the number of cycles in the two outputs is comparable. This in turn requires that the natural frequencies of the two oscillators be close to one another. The second assumption is that to within a scale factor, the integral of the squared spectrum of the oscillator response excited by an arbitrary ground acceleration is equal to the integral of the squared spectrum of the oscillator impulse response; in other words, the excitation spectrum is "flat" enough in the passband of the oscillator to be factored out of the response integral. With this assumption, Parseval's theorem can be used to show that

$$A_{\text{rms}} \sim (T^3/\zeta)^{1/2} \tag{3}$$

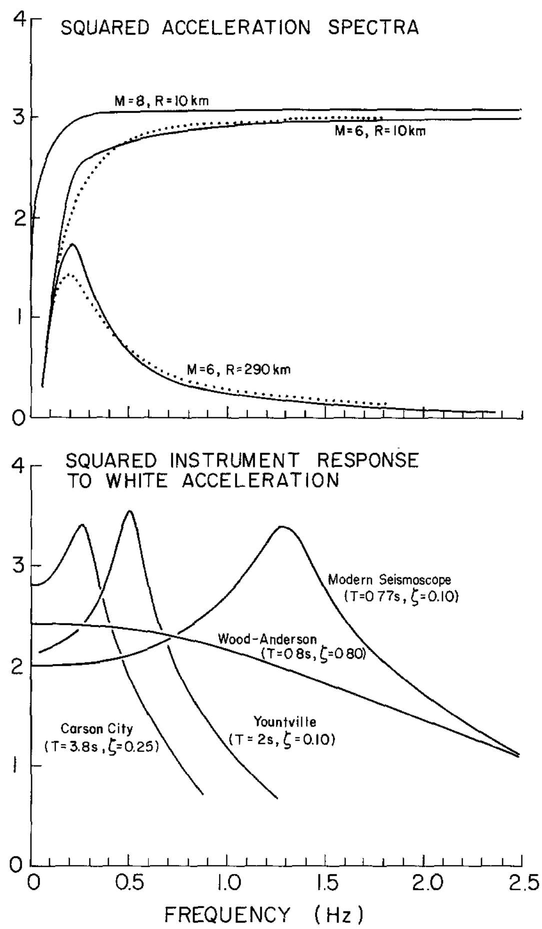

FIG. 1. Squared acceleration spectra (*top*) and squared instrument response to white acceleration (*bottom*). Acceleration spectra are from Joyner's (1984) model, with a critical earthquake of $M = 7$. Dotted curves are for Brune's (1970) scaling model. A stress parameter of 100 bars and a frequency-dependent Q, as given in the text, were assumed. The solid curves in the top and bottom portions of the figures have been arbitrarily adjusted vertically; only relative shapes are of interest. Squared spectra and the linear abscissa were chosen to better assess the merits of the assumption behind equation (1) in the text that to within a scaling factor, the integral of the squared spectrum of the oscillator response (obtained by adding the curves in the top and bottom portions of the figure) equals the integral of the squared oscillator impulse response (from the bottom portion of the figure).

and equation (1) follows. As Figure 1 shows, the validity of the second assumption will depend on the period and damping of the oscillator and the magnitude and distance of the earthquake. A qualitative analysis of Figure 1 suggests that equation (3) gives a better approximation of A_{rms} for a modern seismoscope than for a longer period seismoscope, both excited by a $M = 6$ earthquake at 10 km. Furthermore, at close distances the larger the earthquake, the better the approximation. At large distances the absorption of seismic energy distorts the spectrum to such an extent that the second assumption behind equation (3) may not hold for any size earthquake.

A second application of seismoscope records is to estimate peak horizontal ground velocity. If equation (1) is valid, then the following empirical relation

$$v_{\max} = 0.77 A_{wa} \tag{4}$$

between peak horizontal velocity ($v_{\max}$) and Wood-Anderson response (A_{wa}) (Boore, 1980, 1983) can be used to eliminate A_{wa}, resulting in a relation beween peak velocity and seismoscope response. For a modern seismoscope with resonance frequency of 1.3 Hz and a standard Wood-Anderson instrument ($T = 0.8$s, $\zeta = 0.8$, $V = 2800$), the predicted relation is

$$v_{\max} = 8.1 S_{d_{10}} \tag{5}$$

where $v_{\max}$ is in centimeters/second and, in Jennings and Kanamori's notation, $S_{d_{10}}$ is the maximum response in cm of the seismoscope reduced to the equivalent motion of a unit gain, 10 per cent damped oscillator with resonant period equal to that of the seismoscope [see equation (14) in Jennings and Kanamori, 1979]. The validity of equation (5) depends on that of the two relations from which it was derived. As shown earlier, various objections to the first relation—equation (1)—can be raised, and Boore (1983) showed that at a fixed distance, the second relation—equation (4)—is a good approximation over only a limited magnitude range (which happened to coincide with the magnitudes of the earthquakes providing the data upon which the empirical correlation was made). The purpose of this paper is to make quantitative assessments of the assumptions used in deriving equations (1) and (5).

Methods

Tests of the assumptions have been made by computing the peak ground velocity and the responses of the Wood-Anderson instrument and the various seismoscopes to earthquakes of specified moment magnitudes and distances, using a recently developed method for the stochastic simulation of the motions from a specific source model (Boore, 1983). The method is based on some results from random vibration theory that use various spectral moments of the squared ground motion and instrument responses to predict the mean values of the peak motions corresponding to the spectra. The results have been checked with time domain simulations in which a time sequence of windowed, random Gaussian noise is filtered so that the amplitude spectrum agrees, on the average over an ensemble, with the specified spectra. Details of the random vibration method used here are as given in Boore (1983), except for modifications related to the choice of oscillator response (to be described in a future paper by myself and W. Joyner). The seismological basis of the method lies in the particular shape and magnitude scaling used for the spectra. In the previous paper just referenced, I found that the ω-square model of Brune (1970, 1971) with the addition of a high-cut filter (at 15 Hz) and a constant stress parameter of 100 bars gives a good fit to many observed measures of strong ground motion. A possible objection to using the Brune spectral-scaling model in this paper is that it would be applied to cases, such as the 1906 San Francisco earthquake, in which similarity of the source geometry is no longer valid. For this reason, the majority of the results in this paper are for a recently devised two-corner spectral-scaling model by Joyner (1984) in which similarity holds for earthquakes less than a critical size, but breaks down for larger events (where the higher frequency corner, which may be thought of as being related to fault width, is held constant). When compared with observed data, most of which come from earthquakes for which similarity is a good assumption, the model gives a fit comparable to that of the Brune scaling model (Figure 1 shows acceleration spectra for a $M = 6$ earthquake for both models).

In Boore (1983), I was concerned with the magnitude scaling of ground motions at a close distance to the source; anelastic absorption was not an important factor. In this paper, however, I will be predicting shear wave motions as far as 290 km from the fault. As a provisional model, I have assumed r^{-1} geometrical spreading

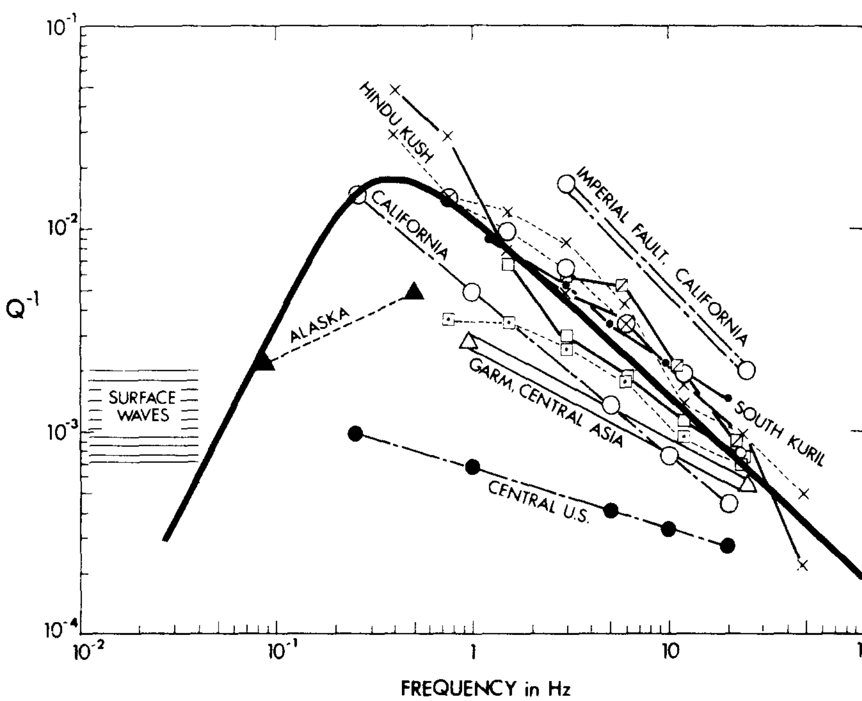

FIG. 2. Observed inverse shear-wave Q versus frequency from Aki (published by Cormier, 1982) and the attenuation function given by equation (6) in the text (heavy line).

and a frequency dependent Q. The specified Q function—

$$Q^{-1} = 0.034 \frac{(f/0.3)^2}{1 + (f/0.3)^{2.9}} \tag{6}$$

—is consistent with frequency dependent Q measurements (summarized in Figure 2). The high frequency behavior is also consistent with analyses of response spectral attenuation by Joyner and Boore (1982).

Results

Estimates of M_L. The results for the modern seismoscope are shown in Figure 3. The solid lines are the new predictions of the ratio of Wood-Anderson to seismoscope response; the dashed line is the value given by the relation [equation (1)] used by

Jennings and Kanamori (1979). The logarithmic ordinate in the figure was chosen so that the difference in M_L stemming from the two estimates of the ratio of Wood-Anderson to seismoscope response could be read directly as the difference in logarithmic units between the two curves. The relation given by equation (1) leads to an overestimate of M_L if the dashed line is above the solid line and vice-versa. The figure predicts that the relation gives an adequate prediction of the Wood-Anderson response—and thus M_L—for earthquakes within at least 50 km of the fault. At larger distances, however, it can lead to systematic underestimates of as much as 0.3 units (but modern seismoscopes might not produce a discernable signal at these larger distances).

The discrepancies between the new predictions and those based on equation (1) are even more pronounced for the instruments that recorded the 1906 earthquake (Figure 4), at least for moment magnitudes less than about 7.5. This is understandable, for the natural periods of the older instruments are greater than those of modern seismoscopes. The increasing discrepancy as magnitude diminishes is due to the proximity of the corner frequency in the acceleration spectrum to the natural

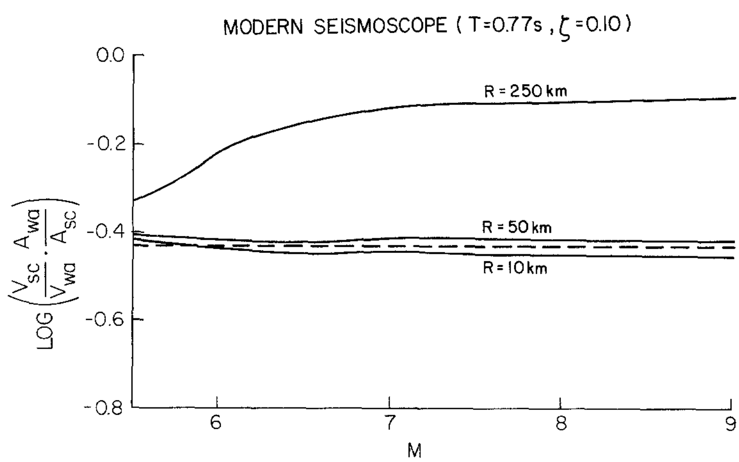

FIG. 3. Ratio of peak Wood-Anderson and modern seismoscope responses, normalized by static magnifications, as a function of moment magnitude. Solid lines from random vibration theory; dashed line from equation (1) of this paper. See text for details of the spectral model used in the calculations.

frequencies of the instruments; the actual source spectrum violates the assumption of a white spectrum for frequencies below the corner frequency. The results in Figure 4 predict that using equation (1) would lead to an underestimate of M_L from the Yountville record and an overestimate from the Carson City record. The 1906 earthquake probably had a moment magnitude greater than about 7.5 (Thatcher, 1975, whose estimate of moment gives a moment magnitude of 7.7), so the correction to the M_L estimated by Jennings and Kanamori would increase M_L from Yountville by about 0.1 units (from their best estimate of 6.3 to 6.4) and decrease the Carson City estimate by about 0.2 units (from 7.2 to 7.0). A large portion of the remaining spread may be due to local geologic site conditions. The Carson City instrument was located in the middle of an alluvium filled valley; the Yountville instrument was at the edge of a valley, adjacent to volcanic bedrock. In an analysis of response spectra as a function of geologic site conditions, magnitudes, and distance, Joyner and Boore (1982) found that at periods of several seconds, 5 per cent damped response spectra at sites underlain by more than 5 m of soil were about 0.3 logarithmic units larger than spectra recorded at rock sites. This suggests that the range in M_L estimates for the 1906 San Francisco earthquake should be reduced by another 0.3 units. If the standard Richter magnitude refers to an average of rock and soil recordings, it might be appropriate to distribute the 0.3 units equally between the two stations, yielding M_L estimates of about 6.5 to 6.8. These estimates are lower than Jennings and Kanamori's final estimates of $6\frac{3}{4}$ to 7.

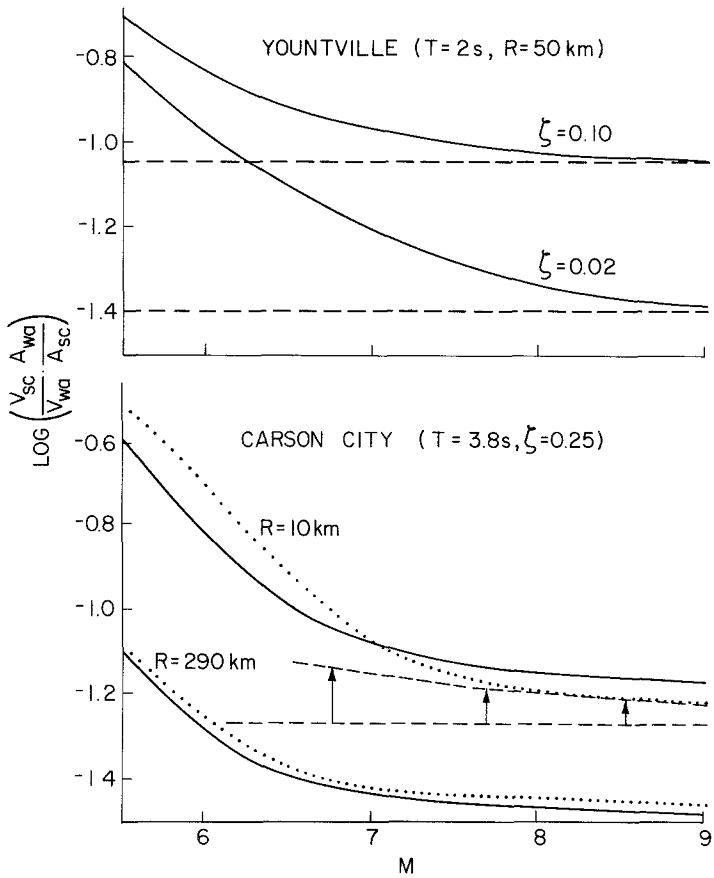

FIG. 4. Ratio of responses from Wood-Anderson instrument and seismoscopes that recorded the 1906 San Francisco earthquake. The two damping values for the Yountville seismoscope span the range estimated by Jennings and Kanamori (1979) for that instrument. Distances used were 50 and 290 km for the Yountville and Carson City recordings, respectively. For Carson City, the ratio was also computed for a close distance (10 km) as well as the fault-station distance used by Jennings and Kanamori in order to see the influence of distance (in addition to the corner frequency effect manifested in the magnitude dependence). Dotted lines are from Brune's (1970) spectral scaling model with a constant stress parameter (see Boore, 1983, for details). The horizontal dashed lines are from the relation used by Jennings and Kanamori to estimate Wood-Anderson response [equation (1) in the text]. The distance between the dashed and solid or dotted curves is a direct measure of the error in M_L from using equation (1). The arrows show how equation (1) is modified if a correction is made for the number of cycles of oscillator response (see text).

To see the sensitivity of the results to the spectral scaling model, the simulations of the Wood-Anderson and Carson City seismoscope responses have been repeated for the Brune scaling model used by Boore (1983). The results, shown as the dotted lines in the lower portion of Figure 4, have the widest divergence for magnitudes around 6. This is easily explained. As seen in Figure 1, for this magnitude the largest difference in acceleration spectra for the two models occurs in a frequency range that includes the peak response of the Carson City seismoscope.

As discussed earlier, the discrepancy between equation (1) and the simulation results is primarily due to the assumptions, made in deriving equation (1), that the excitation spectrum is essentially "flat" and that the ratios of rms responses for two oscillators are the same as the ratio of the peak responses. To isolate the effect of the latter assumption, the relation between A_{max} and A_{rms} given by equation (2) has been used to correct equation (1). The number of extrema are calculated from spectral moments (Boore, 1983); the effect is expected to be largest for the Carson City seismoscope since its resonant frequency differed the most from that of a Wood-Anderson instrument. As shown in the lower part of Figure 4, the correction leads to a substantial decrease in the systematic error for large earthquakes at close distances (for which the acceleration spectra are relatively flat over a wide frequency range).

Estimates of peak horizontal velocity. The predictions of peak velocity given by equation (5)—$v_{max} = 8.1 S_{d_{10}}$—have been checked by simulating v_{max} and $S_{d_{10}}$ for a

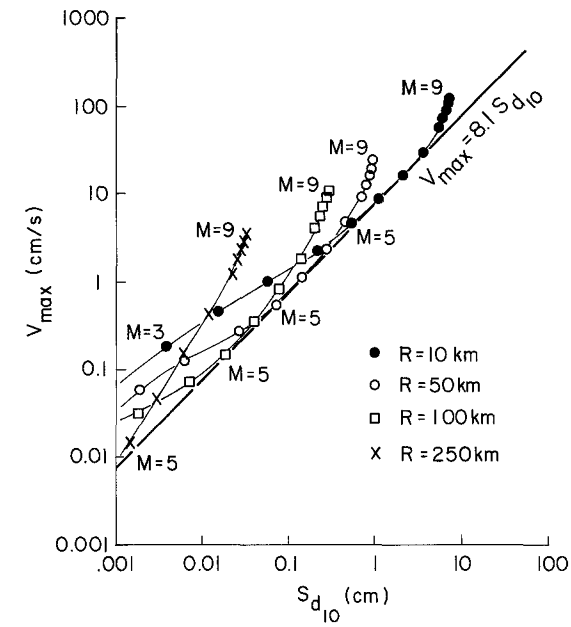

FIG. 5. Correlation between peak horizontal velocity (v_{max}) and the displacement response of a unit gain, 10 per cent damped oscillator with 1.3 Hz resonant frequency ($S_{d_{10}}$). At each of four distances, simulations were made for moment magnitudes ranging in half-unit decrements from $M = 9$. For convenience, the $M = 9$, $M = 5$, and in one case $M = 3$ points have been labeled. The light lines connect points at the same distance. The heavy, straight line ($v_{max} = 8.1 S_{d_{10}}$) is the predicted correlation discussed in the text.

wide range of moment magnitudes and distances of 10, 50, 100, and 250 km. As shown in Figure 5, the correlation given by equation (5) forms a lower bound to predictions of peak velocity from seismoscope response. Equation (5) is quite good for magnitudes ranging from about 4.5 to 6.5 at distances within 100 km, and it underestimates the peak velocity by factors up to 2 or 3 for larger earthquakes. Although not shown, a similar suite of simulations has been done for the Brune spectral model. The major difference is seen at larger magnitudes, where there is less tendency for the $S_{d_{10}}$ values to saturate than in the Joyner model. The overall character of the results remains unchanged.

An interesting application of the simulation method is to the estimation of peak horizontal velocity from the Guatemala City seismoscope record produced by the 1976 Guatemala earthquake. The earthquake had a moment of 2.6×10^{27} dyne cm

(Kanamori and Stewart, 1978), giving a moment magnitude of 7.6 (Hanks and Kanamori, 1979). The closest distance from the station to the fault was about 25 km, and the average seismoscope response was equivalent to an $S_{d_{10}}$ of 4.6 cm (Jennings and Kanamori, 1979). Equation (5) gives a lower bound of 37 cm/sec for the peak velocity. The simulation method predicts $S_{d_{10}} = 2$ cm and $v_{max} = 29$ cm/sec for a rock site. The recording site, however, was underlain by a considerable thickness of soil. One way of accounting for this is to increase the simulations by the empirical soil factors found by Joyner and Boore (1982) for response spectra and peak velocity. These factors are both about 1.6, leading to estimates of $S_{d_{10}} =$ 3.2 cm and $v_{max} = 46$ cm/sec; the difference between the estimated and observed values of $S_{d_{10}}$ is well within the uncertainty in the prediction of a single observation found by Joyner and Boore (1982), giving confidence in the simulated estimate of the peak velocity. The final estimate comes from multiplying v_{max} by the ratio of the observed and estimated seismoscope responses (as a way of incorporating local site amplifications not included in the average factor of 1.6 used above). This gives a final estimate of $v_{max} = 66$ cm/sec for the Guatemala City recording of the 1976 Guatemala earthquake.

Discussion

The results in this paper confirm that modern seismoscope recordings can be used to infer Wood-Anderson instrument response (and thus M_L). They also show that seismoscope recordings can be used to estimate peak horizontal velocity. The simplest procedure uses the relations given by equations (1) and (5). Being independent of earthquake magnitude, source-site distance, and source characteristics, however, these relations should be used with some care. The simulation study described in this paper found that the relation between peak amplitudes on a Wood-Anderson instrument and on a modern seismoscope [equation (1)] is adequate for earthquakes within 50 km, having moment magnitudes above 5.5. The relation underestimates the Wood-Anderson response by as much as a factor of 2 at a distance of 250 km. The relation is not as good, however, for seismoscopes whose resonant frequencies are lower than for the modern seismoscope. An analysis of the seismoscope recordings of the 1906 San Francisco earthquake, first analyzed by Jennings and Kanamori (1979), has reduced the probable magnitude of the earthquake to M_L with 6.5 to 6.8 from their estimate of $6\frac{3}{4}$ to 7, and more significant, has reduced the spread between the estimates.

The estimate of peak horizontal velocity from the seismoscope recordings again depends on magnitude, distance, and source model, but the relation given by equation (5) seems to give a lower bound for v_{max}. An application to a seismoscope recording of the 1976 Guatemala earthquake ($M = 7.6$) gave a lower bound of 37 cm/sec and a more likely value of 66 cm/sec at about 25 km from the fault.

Acknowledgments

Discussions with Ken Campbell were the catalyst for this research and those with W. Joyner greatly aided in completing it.

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Manuscript received 22 March 1983