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Comment and Reply

Comment on "Earthquake Source Spectra in Eastern North America" by R. A. W. Haddon

by G. M. Atkinson, D. M. Boore, and J. Boatwright

Haddon (1996) proposes a theoretical model for earthquake source spectra in eastern North America (ENA), based on certain assumptions regarding the rupture process. He shows that if ENA earthquakes are characterized by very high rupture velocities and fractional stress drop, then their high-frequency *S*-wave radiation is strongly enhanced relative to that predicted by the simple Brune (1970) source model. Haddon discusses previous analyses of ENA ground motions, which appear to be at odds with his model, concluding that these analyses contain misinterpretations of both data and theory. He disagrees specifically with the results obtained by Boatwright and Choy (1992), Boore and Atkinson (1992), Atkinson (1993), Boatwright (1994), and Atkinson and Boore (1995). His discussion of our results is mistaken in many points, as we describe in the following.

The Saguenay Earthquake

Is the Saguenay Earthquake a Typical ENA Event?

Haddon asserts that (p. 1308) "The only substantive data that have been adduced to support the conclusion that the Saguenay earthquake is unusual are the teleseismic *P*-wave results obtained by Boatwright and Choy (1992)." This statement does not reflect the weight of analyses and conclusions contributed by many authors. North *et al.* (1989) point out that the Saguenay earthquake had a high value of Nuttli magnitude ($m_{bLg} = 6.5$) relative to moment magnitude ($\mathbf{M} = 5.8$), in comparison to other events such as the 1982 Miramichi earthquake ($m_{bLg} = 5.7$, $\mathbf{M} = 5.5$), and conclude that the earthquake radiated an unusual amount of high-frequency energy; this anomaly in magnitude values was demonstrated explicitly by Boore and Atkinson (1992). North *et al.* also note that the 1988 Saguenay and 1925 Charlevoix events had similar felt areas, despite the fact that the 1925 Charlevoix event was more than a factor of 2 larger in terms of seismic moment. Hanks and Johnston (1992) show that both the felt area and the damage area of the Saguenay earthquake were anomalous, exceeding the average areas for events of this magnitude by more than a factor of 2: they attribute these relatively large MMI levels to an anomalously large stress drop. Atkinson's (1993) analysis of regional seismographic data showed that the high-frequency level of the Fourier spectrum of the Saguenay earthquake, relative to its seismic moment, is large compared to other ENA events of $4 \leqq \mathbf{M} \leqq 6.8$. Hartzell *et al.* (1994) invert the teleseismic *P* waves from the Saguenay earthquake, together with the strong-motion *S*-wave data that Haddon analyzed. They summarize the results from their time-domain inversions of the five largest ENA earthquakes by stating: "Of the events studied, the Saguenay earthquake is unique in terms of its greater depth, spatially concentrated source, and large asperity stress drop." Finally, Atkinson and Boore (1995) showed that the Saguenay earthquake is the only one of eight ENA mainshocks of $4 \leqq \mathbf{M} \leqq 6.8$ that show significant (>1 sigma) positive residuals relative to their ground-motion prediction model.

Haddon's disagreement with the results of Boatwright and Choy (1992) concerns the manner in which teleseismic *P*-wave spectra were combined with regional *S*-wave spectra. Boatwright and Choy obtain a corrected acceleration source spectrum from teleseismic signals that include the *P*, *pP*, and *sP* phases (Fig. 3 of their article): their source spectrum increases with frequency, reaching a value of $(6 \pm 3) \times 10^6$ cm^2/sec at 2 Hz, the limit of the teleseismic passband. Because the acceleration spectrum is still rising with frequency as the 2-Hz passband limit is approached, the 2-Hz spectral level constitutes a lower bound for the high-frequency acceleration spectral level for the earthquake. Because the corner frequency for the event appeared to be outside of the teleseismic passband, Boatwright and Choy used the average *S*-wave spectra from the regional accelerographs to infer the corner frequency (=2.5 Hz), and the high-frequency spectral level [$=(9 \pm 3) \times 10^6$ cm^2/sec].

Haddon considers Boatwright and Choy's (1992) combination of the teleseismic spectra and the spectra from the *S*-wave arrivals on the regional accelerographs to be "invalid," because they do not explicitly include the effect of directivity. In his discussion of Boatwright and Choy's results, Haddon considers the potential effects of directivity on their *P*- and *S*-wave spectra but does not consider a number of other relevant factors, such as the contributions of the *pP* and *sP* phases to the teleseismic *P*-wave spectra, or the trade-off between directivity and radiation pattern for the regional *S*-wave data. He concludes from his models for the Saguenay earthquake (e.g., based on a rupture velocity of 0.95 β) that the regional *S*-wave observations have been strongly enhanced by directivity, such that *S*-wave estimates of source–spectral level should exceed the *P*-wave estimates by factors of 2 to 6. He argues that Boatwright and Choy's teleseismic P-wave spectrum for Saguenay should be reduced by this amount at high frequencies, which would make the high-frequency spectral level of this event comparable to those of the other intraplate events.

In rebuttal, we point out that the source spectrum that Boatwright and Choy derive from the regional S waves *overlaps* the source spectrum derived from the teleseismic P waves in the frequency band from 1 to 2 Hz (which is the passband covered by both). Haddon's interpretation requires that there be a clear mismatch in source spectral amplitude between these independent estimates. Furthermore, it is clear from the teleseismic P data alone that the source–spectral amplitudes of the Saguenay earthquake are anomalous, relative to those of other intraplate events.

To summarize, we concur with the conclusion reached by North *et al.* (1989), Boatwright and Choy (1992), Boore and Atkinson (1992), Hanks and Johnston (1992), Atkinson (1993), Hartzell (1994), and Atkinson and Boore (1995): The Saguenay earthquake produced high-frequency ground motions that are larger than is typical for ENA events of this moment magnitude.

How Should the Saguenay Event be Weighted?

In Haddon's (1996) discussion of the Saguenay earthquake, he states (p. 1301): "Atkinson and Boore (1995) have assumed, however, that such events are atypical, and they have weighed the data from such events accordingly." This statement does not accurately represent the referenced article. Atkinson and Boore *concluded* that the Saguenay earthquake was atypical, due to the weight of evidence discussed above: Nevertheless, they did not weigh the data accordingly. In evaluating their ground-motion predictions, Atkinson and Boore (1995) gave *equal weight* to all mainshock events of $4.0 \leqq \mathbf{M} \leqq 6.8$, including Saguenay; there are eight such events in the database of Atkinson and Boore (1995). This is appropriate, since there is no reason to believe that another event with characteristics similar to the Saguenay event will not occur in the future. Each past *event* should be considered a single sample of the range of possible future events; note that this argument does not necessarily apply to each *record,* since some events may be over-represented in terms of number of records.

Uncertainty in Attenuation

Haddon (1996) states (pp. 1304–1305): "Most previous analyses of S-wave spectra and attenuation (Q), for eastern North America earthquakes, have simply assumed the Brune spectral model to provide an adequate representation for small magnitude ($\mathbf{M} < 4$) earthquakes (see, e.g., Hasegawa, 1974; Atkinson, 1993; Boatwright, 1994)." He then shows that, if his theoretical source model is correct, the assumption of the Brune source model in a combined regression for source and attenuation parameters could bias the attenuation estimates: The regression would compensate for the misfit source amplitudes by overestimating the anelastic attenuation. Based on the source spectral shape that he expects from his model, Haddon concludes (p. 1312) "current estimates of Q depending on this assumption (i.e., Brune model assumption) are likely to be significantly too large for high frequencies."

Haddon's proposed bias cannot affect the source spectra obtained by Atkinson (1993) or the attenuation model of Atkinson and Mereu (1992), derived from the same dataset. These regressions do not assume a specific source model shape. The results are not predicated on the Brune source model or any other source spectral model.

Atkinson and Mereu (1992) performed multivariate regression analyses of Fourier spectra computed for a window containing all significant S motion (e.g., direct *S, SmS, Sn, Lg*); this constitutes the wave train of interest in engineering applications. The database for the regression analyses was comprised of 1000 digital records, well distributed in distance over the range from 10 to 1000 km. The database was carefully screened to ensure adequate signal-to-noise ratio and adequate resolution (i.e., sufficient number of digital counts) for each record. The database of Atkinson and Mereu was regressed, frequency-by-frequency, to determine the regional attenuation and its uncertainty, using standard statistical techniques [including the use of an L1 norm to minimize the influence of outlying data points, as suggested by Press *et al.* (1986)]: They quantitatively examined such issues as the trade-off between geometric spreading and Q, and evaluated the uncertainty in both the attenuation of spectral amplitudes with distance, and of the source amplitudes. The evaluations of the uncertainty in attenuation and source amplitudes were based on Monte Carlo simulation techniques (Press *et al.,* 1986), using simulated datasets having the same distribution in distance as the study database, and the same random variability. The attenuation analyses showed that a strong trade-off exists between the geometric spreading and Q values, but the overall rate of decay of spectral amplitudes is highly constrained due to the large database, resulting in relatively low uncertainty on near-source spectral amplitudes for most events.

Boatwright (1994) performed a regression analysis of the Atkinson and Mereu (1992) dataset using an entirely different regression technique. Unlike the studies by Atkinson and Mereu (1992) and Atkinson (1993), the regression by Boatwright did include the assumption of a Brune source model. Both Atkinson and Mereu (1992) and Boatwright (1994) determined that, when a geometric spreading coefficient of 1.0 is assumed, a frequency-independent Q of 2000 is obtained.

Atkinson and Mereu's preferred attenuation model features a trilinear attenuation with a geometric attenuation coefficient of 0.5 at regional distances, which has an associated frequency-dependent Q given by $Q = 680 f^{0.36}$. Although this may at first glance appear to be very different from an R^{-1} model with constant $Q = 2000$, there is actually little difference between these two models in the overall rate of decay of spectral amplitudes, over the frequency range from 1 to 10 Hz, as illustrated in Figure 1. This shows how the overall rate of attenuation can be well constrained, despite the limited ability of the data to distinguish between slope (geometric coefficient) and curvature (Q). Figure 1 also shows the implications, for the decay of spectral amplitudes, of replacing Boatwright's $Q = 2000$ estimate with the "somewhat arbitrary" (p. 1307) value of $Q = 1350$, which Haddon uses in constructing his Figures 6 and 7.

If Boatwright's Q result is an artifact of inappropriately assuming a Brune source model, as Haddon claims, then why would Atkinson and Mereu (1992) get exactly the same result, with no Brune source model assumption? According to Haddon's logic, Atkinson and Mereu (1992) should have obtained $Q = 1350$, for an assumed geometric spreading coefficient of 1.0, while Boatwright (1994) obtained $Q = 2000$. The explanation as to why Atkinson and Mereu (1992) and Boatwright (1994) obtained the same result with differing regression models is simple:

- The Brune source model assumption had no perceptible effect on Boatwright's attenuation result, because most of the events match the Brune model shape reasonably well, as shown by Atkinson (1993).
- The rate of decay of spectral amplitudes is robust due to the large volume of data; there are unresolved trade-offs between geometric spreading and Q, but any reasonable regression of this dataset will reproduce the overall decay rates obtained by Atkinson and Mereu (1992) and Boatwright (1994).

Figure 2 examines the implications of Haddon's explanation as to how the source spectra of small events might be biased by attenuation uncertainty, in light of the attenuation database. Normalized spectral amplitudes are plotted for a frequency of 10 Hz, for the attenuation database used by Atkinson and Mereu (1992) and Boatwright (1994). The normalization of amplitudes is accomplished by subtracting, from each recorded amplitude, the mean source term for the corresponding event and the mean site term for the corresponding station, as determined by the regression. This simply shifts the attenuation curve for each event up or down to a common source level of 0.0, without affecting either the attenuation of amplitudes with distance or the intraevent variability. Figure 2 demonstrates that the mean overall decay rate of spectral amplitudes in ENA is highly constrained. Figure 2 also shows the trilinear attenuation form of Atkinson and Mereu (1992), as used by Atkinson (1993) to determine the source spectral amplitudes; these source amplitudes match those obtained by Boatwright (1994) for his attenuation model of R^{-1} with $Q = 2000$ (also shown in the figure). Haddon suggests that the difference between Boatwright's R^{-1} model with $Q = 2000$ and an equivalent model with $Q = 1350$ represents "relatively small uncertainties in attenuation (Q)" (p. 1307). We observe in Figure 2 that this uncertainty represents a factor of 4 on spectral

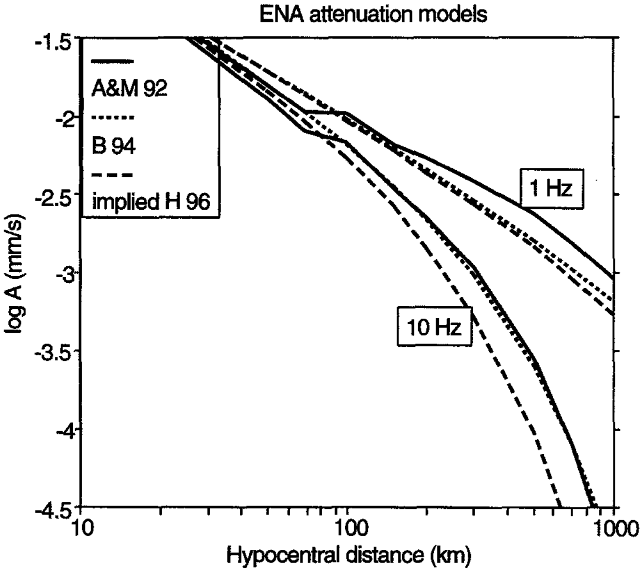

Figure 1. Comparison of rate of decay of spectral amplitudes for the trilinear attenuation model of Atkinson and Mereu (A&M92), in comparison to the linear $Q = 2000$ model of Boatwright (B94), for frequencies of 1 and 10 Hz. The attenuation model discussed by Haddon ($Q = 1350$) is also shown.

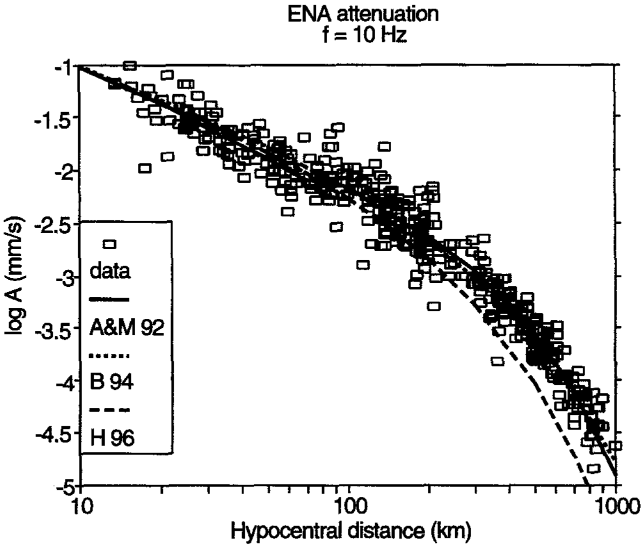

Figure 2. Attenuation of 10-Hz Fourier spectral amplitudes in ENA. Plus symbols show normalized ECTN amplitudes, obtained by subtracting the regression source term for the event and the regression site term for the station (shifts curves up or down to common level). Solid line shows trilinear attenuation form of Atkinson and Mereu (1992); dotted line shows that the Boatwright (1994) attenuation result is difficult to distinguish from the Atkinson and Mereu attenuation. Dashed line shows attenuation postulated by Haddon (1996).

amplitudes at 800 km; we also observe that this amount of uncertainty is not present in the mean decay rate of spectral amplitudes. (Note the distinction between uncertainty on the mean and uncertainty on the amplitude of an individual data point.)

Because the uncertainty in the mean decay rate of spectral amplitudes is low, the uncertainties in source spectral amplitudes are generally low—about 0.1 log units, depending on the distance distribution of the stations recording the particular event, as well as the details of the variability for that event. To illustrate for a specific event, Figure 3 shows the uncertainty in source spectrum for the 1983 Goodnow, New York, earthquake of **M** 5.0. On the figure, the 90% confidence limits on the source spectrum are shown, as determined from Monte Carlo error analysis of the regressions of the ENA seismographic database (see Atkinson and Mereu, 1992, for details). Also shown in the figure are the source spectral models of Atkinson (1993) and Haddon (1996) for an event of this seismic moment, as well as the Brune (1970) source model for a stress parameter of 80 bars (chosen to match the high-frequency level of this event). In Figure 3, it is clear that the uncertainty in the source spectrum, due to uncertainty in attenuation, is too small to accommodate Haddon's speculation that the source amplitudes continue to rise significantly from 5 to 15 Hz.

An empirical study of high-frequency spectral shapes (Atkinson, 1996), based on over a dozen small-to-moderate events, showed that ENA source spectral amplitudes are essentially flat for frequencies greater than two times the Brune corner frequency, up to frequencies of at least 30 Hz. Figure 3 provides one example of this trend. Another, more specific, example of the high-frequency shape for a small ENA earthquake is provided by Hough *et al.* (1989), for an event of **M** 3.1 that occurred near Massena, New York. Using data recorded very near the source (9 km), their analysis clearly demonstrates that the acceleration spectrum is flat in the frequency band from 10 to 30 Hz, then decays slightly for higher frequencies; they also conclude it is well described by a simple omega-squared (Brune) model. In general, then, empirical data on the shape of the high-frequency spectrum for small events do not support the theoretical model upon which Haddon bases his conclusions regarding attenuation. Finally, we point out that Haddon does not present any data demonstrating the attenuation of spectral amplitudes with distance.

Note: We thank Haddon for bringing to our attention an instrument-response error in some of the records used by Atkinson and Mereu and Boatwright, which occurred as a result of changes in station instrumentation near the end of the time period of the data. On investigation of this problem, we determined that 6 of the 1000 records in the Atkinson and Mereu database used an incorrect instrument response. Repeating the 1992 regressions with the corrected data values does not significantly affect any of the previous results for attenuation or site terms; the source spectra remain unchanged for all events with the exception of the Mont Laurier event, which now has lower source spectral amplitudes, by about a factor of 2.

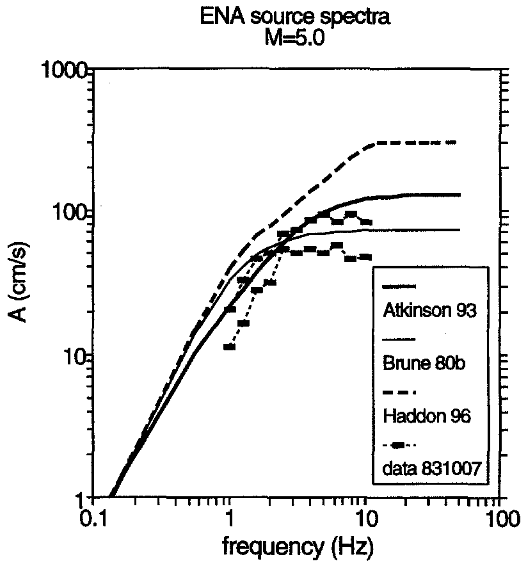

Figure 3. Source spectrum for the 1983 Goodnow, New York, earthquake of **M** = 5.0. Horizontal bars connected by light dotted lines show the 90% confidence limits on the source spectrum, accounting for uncertainty in attenuation. Heavy solid line shows source spectral model of Atkinson (1993) for **M** = 5; light solid line shows Brune (1970) single-corner frequency model for a stress parameter of 80 bars. Dashed line shows Haddon (1996) source model for **M** = 5.

Spectral Ratio Results of Haddon

Haddon (1996) supports his model by reference to the inferred shape of the source spectrum for the 1988 Saguenay and 1990 Mont Laurier earthquakes, as determined from spectral ratios of the aftershocks of these events to the mainshock. Inspection of Haddon's Figure 5 shows that the source spectrum for the Mont Laurier event appears to be flattening near 10 Hz, then rises in the frequency range from 10 to 20 Hz; thus his conclusions about the high-frequency shape of the Mont Laurier source spectrum are critically dependent on the spectral ratios in the frequency range from 10 to 20 Hz. Haddon's analysis of the spectral ratios for Mont Laurier (Haddon, 1996b) are based on aftershocks with magnitudes (m_{bLg}) in the range from 2.5 to 3.2, recorded on the Eastern Canada Telemetered Network (ECTN), at distances up to 470 km. There are two reasons why the spectral ratios for the Mont Laurier earthquake are unreliable in the 10 to 20-Hz frequency band:

1. The corner frequency of some of the aftershocks may be near or within the 10- to 20-Hz frequency band, causing the acceleration spectral ratios to rise within this frequency range. Haddon states that (p. 1305) "The source spectral amplitudes of the denominator earthquakes are

therefore flat below at least 10 Hz." Based on this statement, source spectra could theoretically be obtained from the ratios "below at least 10 Hz," but not within the 10- to 20-Hz band.

2. The high-frequency data from the ECTN network for events as small as m_{bLg} 2.5, at distances of several hundred kilometers, are not reliable for determining spectral amplitudes at frequencies above the 10-Hz corner frequency of the instruments. Haddon does not state what analyses of signal-to-noise ratio, if any, were performed. Even if there were no noise, however, the high-frequency spectral amplitudes of these weak signals would be unreliable due to limited resolution.

We demonstrate the resolution problem for small events at high frequencies, using the ECTN Mark II instrument as an example: high-frequency problems are even more pronounced for the Mark III instruments. To check the reliability of computed spectra for a given magnitude and distance, we stochastically simulate an event. The simulation is based on the methodology of Boore (1983), with the underlying spectrum specified by a Brune source model for a stress parameter of 180 bars, attenuated according to the results of Atkinson and Mereu (1992) and Boatwright (1994). Note that the underlying spectrum is unimportant in this exercise, since we are not testing the validity of the spectral model, only the ability of the ECTN instruments to recover the specified input spectrum. We take the simulated time series, after convolution with the instrument response, and convert it to digital counts as would be recorded by an ECTN instrument (1 digital count = 10 nm/sec); then we take the spectrum of this record and remove the ECTN instrument response. Ideally, we should obtain a computed spectrum that closely matches the input spectrum for the stochastic simulation. Of course, the computed spectrum will have a stochastic character while the input spectrum is smooth. In Figure 4a, we demonstrate that computed spectra from Mark II ECTN records are reliable for frequencies from 1 to 25 Hz for strong signals; the specific example we use is that of an event of **M** 4.0 (m_{bLg} 4.5) at R = 300 km. By contrast, weak signals, giving of the order of 100 digital counts or less, are unreliable for the computation of spectra over much of the frequency range of interest. This is demonstrated in Figure 4b, for the specific example of an event of **M** 2.0 (m_{bLg} 2.5) at 300 km. We conclude that Haddon's source spectra for the Mont Laurier event, as obtained from spectral ratios based on its aftershocks, are unreliable for frequencies above 10 Hz.

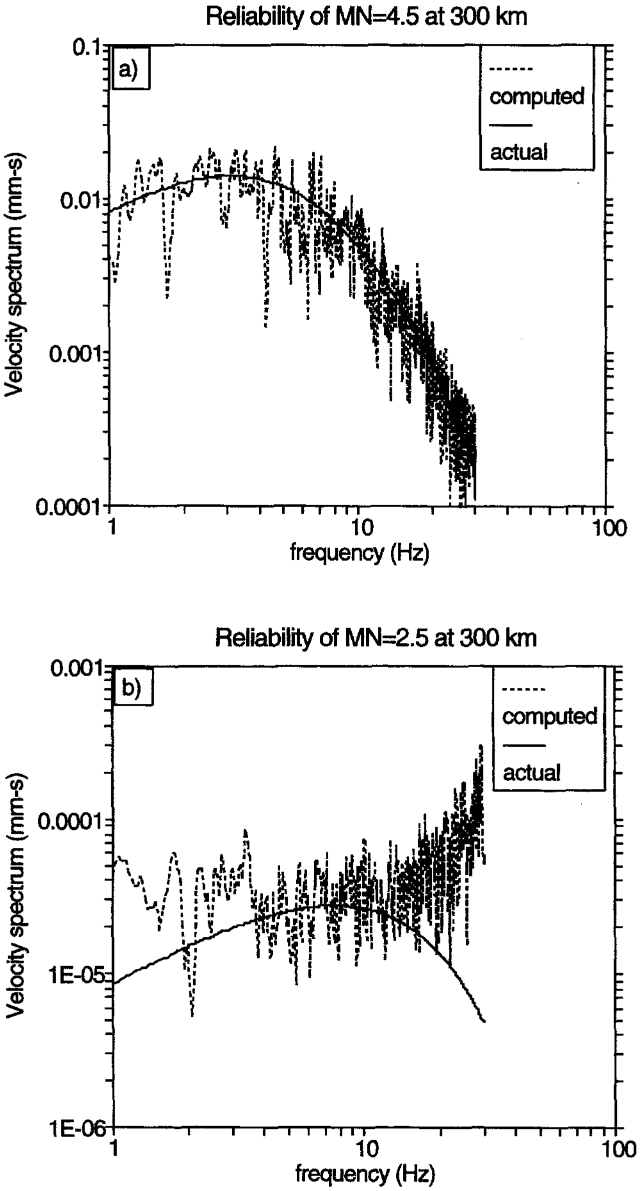

Figure 4. Reliability of ECTN Mark II instrument for recovering spectral amplitudes. (a) Computed spectrum for an event of m_{bLg} 4.5 at 300 km (dashed line), stochastically generated for the underlying input spectrum given by the solid line. (b) Computed spectrum for an event of m_{bLg} 2.5 at 300 km (dashed line), stochastically generated for the underlying input spectrum given by the solid line. Spectral amplitudes of weak signals are unreliable at high and low frequencies, due to poor instrument resolution.

The Role of Stress Drop in Ground-Motion Modeling

Haddon (1996) misinterprets the role of stress drop in our ground-motion modeling and its relationship with physical processes. The stress parameter is in essence a scaling parameter for the high-frequency spectral level. It has no clear physical meaning, although it takes on meaning in the context of the stochastic model, due to its role in controlling high-frequency amplitude levels [see Atkinson and Beresnev (1997) for a discussion of this issue]. For ground-motion modeling, it is irrelevant whether a given high-frequency spectral level is described by a "complete Brune stress drop" of 500 bars, or a "fractional Brune stress drop" of 100 bars; both of these are artificial constructs that do not describe the complex and heterogeneous physical processes of stress release on the surface of a fault. We disagree with Haddon's statement (p. 1300) that ''Observed characteristics of S-wave spectra in ENA are fully explained as simple consequences of directivity effects entailed by the classical crack rupture model, with normal effective stresses of the order of 100 bars and fractional stress drop.'' Real ruptures are much more complicated than this, particularly for large extended fault sources.

In summary, a significant portion of Haddon's article is based on inaccurate interpretations of previous analyses. Further understanding of the range of uncertainty in ENA ground motions requires a balanced approach, which recognizes that there is more than one possible explanation for the range of observed ENA ground motions.

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