Errata to meaning of cosine terms in equation (1) of Boore (2013)

In my paper (Boore, 2013) I state "The cosine term [in equation 1, shown below] accounts for the projection of the cross sectional area in the ray tube onto the horizontal plane representing the free surface."

$$A = \left(\frac{Z_R}{\bar{Z}}\right)^{1/2} \left(\frac{\cos\Theta_R}{\cos\bar{\Theta}}\right)^{1/2} = \left(\frac{\rho_R V_R}{\bar{\rho}\bar{V}}\right)^{1/2} \left(\frac{\cos\Theta_R}{\cos\bar{\Theta}}\right)^{1/2} \quad (1)$$

The statement above the equation is incorrect: the equation does not account for the projection onto a horizontal plane, but rather the change in amplitude due to a change in the cross-sectional area of the ray tube. The equation comes from Joyner et al. (1981), and they state:

> tion, the energy along a tube of rays is constant, and the amplitude is inversely proportional to the square root of the product of the density and the propagation velocity. If we include a correction for the change in cross sectional area of the wave tube due to refraction in a medium where velocity is a function of depth we obtain an expression for the amplification ratio at a soil site
>
> $$A = \left(\frac{\rho_R V_R}{\rho_S V_S}\right)^{1/2} \left(\frac{\cos i_R}{\cos i_S}\right)^{1/2}$$
>
> where V_R and V_S are the near-surface velocities in rock and soil respectively, ρ_R and ρ_S are the densities, and i_R and i_S are the angles of incidence. The factor involving
>
> the angles of incidence can be neglected in cases such as this where the angle of incidence in rock is not large. If the angle of incidence is neglected the expression reduces to one derived by Wiggins (1964) on a somewhat different basis. To use the

Consider plane waves in a constant velocity rock halfspace impinging on the bottom of a constant velocity soil halfspace. Here is a sketch:

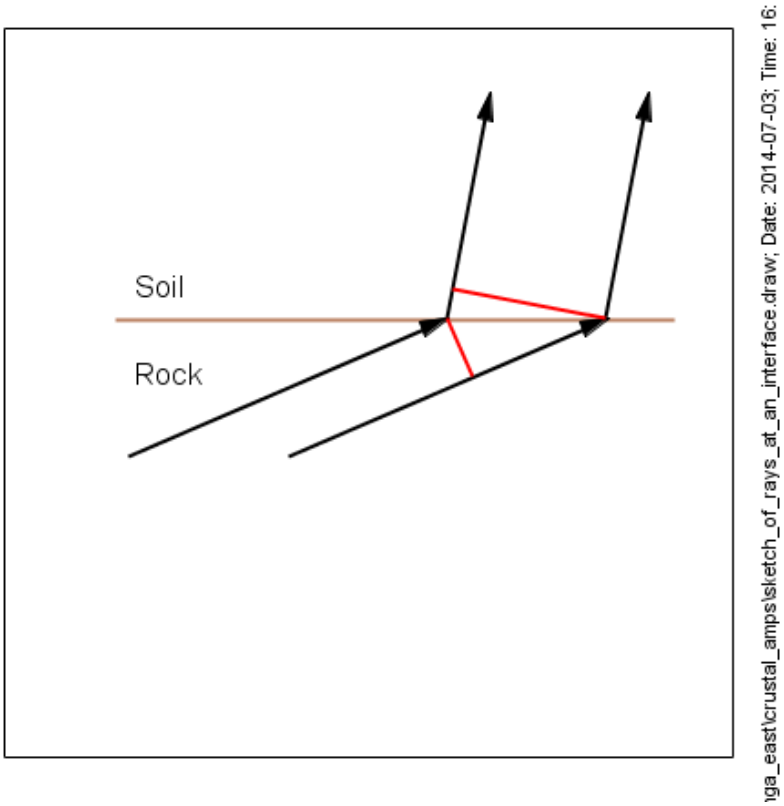

Figure 1.

Note the change in the cross section of the ray tube. The cosine term accounts for this change in the cross section in this projection. Assuming that the ray refraction due to the changing velocity will not produce a refraction out of the plane of the figure, the ratio of cross sectional areas of the ray tube is given by the ratio of the cosines of the angles of incidence. Note that the formulation is that the energy flux per unit time through the perpendicular cross-section of the ray tube is constant—this gives equation (1). So even without a change in amplitude due to the impedance changing, the amplitude in the soil will be smaller than the rock because the energy in the ray tube is spread over a larger cross section.

References

Boore, D.M. (2013). The uses and limitations of the square-root impedance method for computing site amplification, *Bull. Seismol. Soc. Am.* **103**, 2356–2368.

Joyner, W. B., R. E. Warrick, and T. E. Fumal (1981). The effect of Quaternary alluvium on strong ground motion in the Coyote Lake, California, earthquake of 1979, *Bull. Seismol. Soc. Am.* **71**, 1333-1349.