JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 89, NO. B7, PAGES 6229–6235, JULY 10, 1984

Moment-Magnitude Relations in Theory and Practice

THOMAS C. HANKS AND DAVID M. BOORE

U.S. Geological Survey

The observation that motivates this study is the difference in c values in moment-magnitude relations of the form $\log M_0 = cM_L + d$ between central and southern California. This difference is not at all related to geographical area; rather, it results from positive curvature in the $\log M_0 - M_L$ plane and the relatively large number of $M_L < 5$ earthquakes in the central California data set. With the prescription that the far-field shear waves from which M_L is taken be finite-duration, band-limited, white Gaussian noise in acceleration, we can estimate M_L as a function of M_0 alone, by fixing the a_{rms} stress drop at 100 bars and f_{max} at 15 Hz. These model calculations fit available California moment-magnitude data for $0 \lesssim M_L \lesssim 7$, $10^{17} \lesssim M_0 \lesssim 10^{28}$ dyne cm with striking accuracy. This range in source strength is entire: earthquakes with $M_0 \geq 10^{28}$ dyne cm are unlikely to occur in California, and earthquakes with $M_L < 0$ cannot be recorded in California, at least under ordinary conditions of recording earthquakes at ordinary hypocentral depths. More fundamentally, the remarkably good fit of model to data implies that the a_{rms} stress drop of 100 bars (to a factor of 2 or so) is a stable and pervasive feature of all ($M_L \gtrsim 2\frac{1}{2}$) California earthquakes whose spectral corner frequency lies in the "visible" bandwidth, $f_0 \leq f_{max}$.

INTRODUCTION

Moment-magnitude relations have played an important role in earthquake mechanism studies since seismic source parameter determinations became routine in the early 1970's. These empirically defined relations have always been written as a linear relation between $\log M_0$ and M:

$$\log M_0 = cM + d \tag{1}$$

where M_0 is seismic moment and M, in general, can be any magnitude but in practice is usually M_L (local magnitude) or M_s (surface-wave magnitude).

At the present time, the significance of such relations are twofold. First, any definition of a moment magnitude scale (that is, some moment magnitude **M** determined from $\log M_0$) would, ideally, have coefficients (of the inverse relation) not too different from those in (1) for whatever M is actually in use for the region of interest. Happily enough, this can be arranged. *Hanks and Kanamori* [1979] noted that the moment magnitude

$$\mathbf{M} \equiv \tfrac{2}{3} \log M_0 - 10.7 \tag{2}$$

is identical (in inverse form) to the moment-magnitude relations of *Thatcher and Hanks* [1973] for southern California earthquakes ($3 \lesssim M_L \lesssim 7$), of *Purcaru and Berckhemer* [1978] for a set of global earthquakes ($5 \lesssim M_s \lesssim 7\frac{1}{2}$), and of that implicit in the work of *Kanamori* [1977] for great earthquakes ($M_w \gtrsim 7\frac{1}{2}$).

Second, recent studies of high-frequency strong ground motion as finite-duration, band-limited, white Gaussian noise [*Hanks and McGuire*, 1981; *Boore*, 1983] have indicated that ground motion scaling laws, at least in California, can be reduced to a dependence on a single parameter, M_0. Most of the empirical ground motion studies, however, relate the parameter of interest to some M, and thus the transformation between M_0 and the appropriate M is necessary to equate the theoretical and empirical studies. Ideally, again, some uniformly valid moment magnitude scale would be just the thing.

Even when $c = 1.5$ and $d = 16.0$, for which (1) and (2) are nominally the same, it is worth emphasizing the important differences between (1) and (2). Equation (2) is simply a definition, as any magnitude scale is, with constants that more or less square with observational findings, if one is careful in choosing permissible ranges for the appropriate M's; its virtue is that it is uniformly valid in M_0. Equation (1), however, will always fail for a large enough variation in M_0. This failure is due to the size-dependent frequency characteristics of the source excitation and the finite record bandwidth of any time domain amplitude-based M, phenomena that lead to magnitude and peak acceleration saturation [*Hanks and Kanamori*, 1979].

Observationally, this failure of (1) is expressed by c values that increase with M_L. A number of $\log M_0 - M_L$ relations have now been reported for California earthquakes. These are summarized in Table 1, and a discussion of them forms the next section. Here it suffices to note the large difference in the c value found by *Thatcher and Hanks* [1973] for southern California data compared to those obtained in central California. Is this difference a function of the source region or (as we and *Bakun* [1984] believe) the preponderance of small ($M_L \lesssim 4$) earthquakes contributing to the central California data set? The matter is far from academic, should one be interested in estimating M_0 for an $M_L = 6.5$ earthquake in central California, say for the purposes of ground motion estimation mentioned two paragraphs above. The relations of *Bakun and Lindh* [1977], *Archuleta et al.* [1982] ($3.5 \lesssim M_L \lesssim 6.2$), and *Bolt and Herraiz* [1983] yield 0.8, 0.4, and 1.3×10^{25} dyne cm, respectively, whereas the relation of *Thatcher and Hanks* [1973] yields 6.3×10^{25} dyne cm, 5–20 times larger.

SEISMIC MOMENT-LOCAL MAGNITUDE RELATIONS

Investigation of these relations begins with *Wyss and Brune* [1968], who give two. The first of these is for the "San Andreas fault." It is formed from analysis of 12 events near Hollister, Parkfield, and Brawley and an additional earthquake in the Gulf of California, the last event being the only one with $M_L \geq 5.0$. This relation is

$$\log M_0 = 1.4M_L + 17.0 \qquad 3 \leq M_L \leq 6$$

Their second relation is for 259 earthquakes throughout the western United States, the M_0 results being obtained from the

Paper number 4B0432.

TABLE 1. Seismic Moment-Local Magnitude Relations for California Earthquakes

Study	Relation	Range of Validity	Number of Events	Number of Events $M_L \geq 5.0$
	Central California			
Johnson and McEvilly [1974][a]	$\log M_0 = (1.16 \pm 0.06)M_L + 17.60 \pm 0.28$	$2.4 \leq M_L \leq 5.1$	13	1
Bakun and Lindh [1977][b]	$\log M_0 = (1.21 \pm 0.03)M_L + 17.02 \pm 0.07$	$10^{17} \leq M_0 \leq 10^{25}$	28	2
Archuleta et al. [1982][c]	$\log M_0 = (0.96 \pm 0.06)M_L + 18.14 \pm 0.23$	$2.9 \leq M_L \leq 6.2$	40	1
	$\log M_0 = (1.05 \pm 0.08)M_L + 17.76 \pm 0.33$	$3.5 \leq M_L \leq 6.2$	29	1
Bolt and Herraiz [1983]	$\log M_0 = (1.11 \pm 0.15)M_L + 17.92 \pm 1.02$	$3 \leq M_L \leq 6.2$	16	6
Fletcher et al. [1984][b]	$\log M_0 = (1.36 \pm 0.22)M_L + 16.78 \pm 1.07$	$4.3 \leq M_L \leq 5.7$	7	2
	$(1.08 \pm 0.14)M_L + 18.00 \pm 0.51$	$2.8 \leq M_L \leq 4.1$	7	N/A
	Southern California			
Thatcher and Hanks [1973]	$\log M_0 = 1.5M_L + 16.0$	$3 \lesssim M_L \lesssim 7$	138	43
Archuleta et al. [1982][d]	$\log M_0 = (1.39 \pm 0.04)M_L + 16.3 \pm 0.17$	same	same	same

[a] Hollister–Bear Valley region.
[b] Oroville aftershocks.
[c] Mammoth Lakes, May–June 1980.
[d] Least squares fit to data of *Thatcher and Hanks* [1973].

AR technique [*Brune et al.*, 1963] calibrated by the 13 earthquakes above,

$$\log M_0 = 1.7M_L + 15.1 \qquad 3 \leq M_L \leq 6$$

We have not included these relations in Table 1, since neither of them fits neatly as "central California" or "southern California," even though we shall shortly conclude that this distinction is immaterial.

There are two noteworthy features of the central California relations in Table 1. The first of these is the small number of $M_L \geq 5.0$ earthquakes that contribute to the data set, either in part or in sum. The situation is even worse than indicated in Table 1, since three $M_L \geq 5.0$ earthquakes have been used more than once. Only nine different $M_L \geq 5.0$ earthquakes are involved in all of these studies, whereas *Thatcher and Hanks* [1973] included 43 $M_L \geq 5.0$ southern California earthquakes. As we shall see shortly, this is the reason for the differences between the central and southern California relations.

The second feature is that none of the central California studies of Table 1 qualifies as a systematic, regional study of the $M_0 - M_L$ relationships of central California earthquakes. Four of these five studies are for very localized source regions, and the study of *Bolt and Herraiz* [1983] is, in effect, one as well, since 10 of the 16 events are taken from *Johnson and McEvilly* [1974]. This feature, however, is of no real consequence, since the very similar relations for four different regions certainly indicates a regional norm—for $M_L < 5.0$.

The recent study of *Bakun* [1984] does qualify as a systematic regional study, at least for $M_L < 5$. He determined seismic moments for 118 events in five separate source regions: Parkfield, San Juan Bautista, Sargent fault, Coyote Lake, and Livermore Valley. Even though his study, as well, works with only five $M_L \geq 5.0$ events (and only 14 $M_L \geq 4.0$ events), *Bakun* [1984] detected a clear change in the c value at $M_L \simeq 3$. In summary form, with ranges of validity, *Bakun* [1984] finds

$$\log M_0 = 1.2M_L + 17 \qquad 1\tfrac{1}{2} \leq M_L \leq 3\tfrac{1}{2} \tag{3a}$$

and

$$\log M_0 = 1.5M_L + 16 \qquad 3 \leq M_L \leq 6\tfrac{1}{2} \tag{3b}$$

Bakun's study underscores the necessity of working with a wide range of M_L. *Thatcher and Hanks* [1973] did not detect a change in the c-value at $M_L \simeq 3\frac{1}{2}$, but only 12 of their 138 earthquakes had $M_L < 3\frac{1}{2}$ and only four with $M_L < 3$. The studies in central California suffer from the opposite problem: the relatively small number of $M_L \geq 5.0$ earthquakes analyzed to date.

Table 2 gives $M_0 - M_L$ data for 18 $M_L \geq 5.0$ earthquakes in central California, for which we know of a quantitative estimate of M_0. These include all of the $M_L \geq 5.0$ earthquakes used in the central California studies of Table 1, as well as nine additional events culled from the literature. These are plotted in Figure 1, together with the $M_0 - M_L$ relations of Table 1 for the given ranges of validity. With the exception of the relation of *Fletcher et al.* [1984] for $4.3 \leq M_L \leq 5.7$, none of the central California relations is a close approximation to the data of Table 2 above $M_L \simeq 5\frac{1}{2}$, stated ranges of validity notwithstanding. The "southern California" relation of *Thatcher and Hanks* [1973], however, is a close approximation to the "central California" earthquakes with $M_L \geq 5$, excepting the two largest. Clearly, as *Bakun* [1984] has found, the log $M_0 - M_L$ data have positive curvature (see also Figure 2). Straight-line fits of equation (1) to various ranges of the data will result in c values increasing with M_L. As straight-line approximations, we can concur with the findings of *Bakun* [1984], equation (3) above with their given ranges of validity. Above $M_L \simeq 6\frac{1}{2}$, however, the c value must be even larger. In the next section, we describe model calculations that recover in detail both the continuous curvature of the log $M_0 - M_L$ observations and their absolute values, allowing us to forego altogether straight-line fits to log $M_0 - M_L$ data, across limited magnitude ranges chosen more or less arbitrarily.

Before proceeding to these calculations, however, several brief statements on the M_0 estimates in Table 2 and Figure 1 are appropriate. First, we have preferred the teleseismic estimates of M_0 for the Mammoth Lakes earthquakes (Table 2) over the locally determined values of *Uhrhammer and Ferguson* [1980] and *Archuleta et al.* [1982]. The teleseismic estimates are typically 2 to 4 times larger than the local determinations, the latter having strongly conditioned the $M_0 - M_L$ relations of *Archuleta et al.* [1982] and *Bolt and Herraiz* [1983]. Second, the Eureka earthquake is hardly a "central California" earthquake, but it is our point of view, on the basis of the "southern California" fit [*Thatcher and Hanks*, 1973] to "central California" earthquakes (Figure 1), that

TABLE 2. Moment-Magnitude (M_L) Data for Central California Earthquakes, $M_L \geq 5.0$

Location	Date	Origin Time	M_L	log M_0, dyne cm	References
San Francisco	April 18, 1906	1312	6.3, 7.0	27.36, 27.94	*Ben-Menahem* [1978], *Boore* [1984], *Jennings and Kanamori* [1979], *Thatcher* [1975]
Parkfield	June 8, 1934	0447	5.6	25.064	*Bakun and McEvilly* [1984]
Parkfield	June 28, 1966	0426	5.6	25.14	*Kanamori and Jennings* [1978], *Tsai and Aki* [1969]
Truckee	Sept. 12, 1966	1641	5.8	24.85	*Burdick* [1977], *Ryall et al.* [1968], *Tsai and Aki* [1970]
Sargent fault	Dec. 18, 1967	1724	5.3	23.5	*Bakun* [1984]
Melendy Ranch	Feb. 24, 1972	1556	5.1	23.49	*Helmberger and Johnson* [1977]
Oroville	Aug. 1, 1975	2020	5.7	24.80	*Langston and Butler* [1976], *Morrison et al.* [1976], Wallace (quoted by *Cohn et al.* [1982])
Oroville	Aug. 2, 1975	2022	5.1	23.52	*Fletcher et al.* [1984]
Oroville	Aug. 2, 1975	2059	5.2	22.58	*Fletcher et al.* [1984]
Coyote Lake	Aug. 6, 1979	1750	6.0	24.66	*Liu and Helmberger* [1983], *Uhrhammer* [1980]
Livermore Valley	Jan. 24, 1980	1900	5.8	24.68	*Bolt et al.* [1981], *Ferguson et al.* [1980] T. Wallace (personal communication, 1983)
Livermore Valley	Jan. 27, 1980	0234	5.4	24.114	*Bolt et al.* [1981], *Ferguson et al.* [1980]
Mammoth Lakes	May 25, 1980	1633	6.1	25.37	*Barker and Langston* [1983], *Given et al.* [1982], *Uhrhammer and Ferguson* [1980]
Mammoth Lakes	May 25, 1980	1649	6.0	24.57	*Uhrhammer and Ferguson* [1980]
Mammoth Lakes	May 25, 1980	1944	6.1	25.11	*Given et al.* [1982], *Uhrhammer and Ferguson* [1980]
Mammoth Lakes	May 27, 1980	1450	6.2	25.03	*Barker and Langston* [1983], *Given et al.* [1982], *Uhrhammer and Ferguson* [1980]
Eureka	Nov. 8, 1980	1027	6.9	27.11	*McKenzie et al.* [1980], *Lay et al.* [1982]
Coalinga	May 2, 1983	2342	6.2	25.7	K. Hutton (personal communication, 1983), H. Kanamori (personal communication, 1983), R. Miller (personal communication, 1983)

source location is of no real consequence. Finally, no good explanation exists for the unusually low M_0 of the $M_L = 5.2$ Oroville aftershock [*Fletcher et al.*, 1984].

MODELING OF THE MOMENT-MAGNITUDE DATA

Figure 2 presents a large number of moment-magnitude data for central California earthquakes. The preponderance comes from *Bakun* ([1984], 118 events), although the results from a number of other studies have also been used (Figure 2 caption). The entries in Table 2 are also included in Figure 2, these being our preferred estimates for these earthquakes.

Model calculations (large solid circles in Figure 2) are obtained from the numerical simulations of *Boore* [1983] according to the prescription of *Hanks and McGuire* [1981] that the far-field shear-wave acceleration be finite-duration, band-limited white Gaussian noise. The duration is the faulting duration T_d beginning with the direct shear-arrival; the bandwidth is determined at low frequencies by the earthquake corner frequency $f_0 = 1/T_d$ and at high frequencies by the source-size-independent cutoff frequency f_{max} [*Hanks*, 1982]. The calculations of *Boore* [1983] generate stochastic realizations of this prescription, as constrained in addition by the related *Brune* [1970, 1971] spectrum and the a_{rms} stress drop $\Delta\sigma$ [*Hanks and McGuire*, 1981]. From these synthetic acceleration time histories, all manner of time domain and spectral amplitudes can be calculated [*Boore*, 1983], including the maximum amplitude on the Wood-Anderson seismogram that is the basis of M_L.

Just as in the work of *Hanks and McGuire* [1981], the independent variables for the *Boore* [1983] calculations are $\Delta\sigma, f_0$, and f_{max}. The finding of *Hanks and McGuire* [1981] and Boore [1983] that $\Delta\sigma$ is constant (namely, 100 bars) allows the $\Delta\sigma$ and f_0 dependences to be condensed to M_0 (e.g., equation (5) below). If, in addition, we take f_{max} to be constant, the model calculations of *Boore* [1983] reduce to a dependence on M_0 alone. In particular, then, M_0 is the independent variable for the calculations in Figure 2, $\Delta\sigma = 100$ bars and $f_{max} = 15$ Hz being fixed parameters, and M_L is the derived quantity.

In view of the many restrictive assumptions forced on the model calculations, we consider their fit to the data in Figure 2 to be surprisingly good, if any allowance at all is made for naturally arising scatter in the observations not related to variable a_{rms} stress drop. At the larger magnitudes, it can be improved somewhat with the finding of *Luco* [1982] that the M_L for large earthquakes obtained from strong motion accelerograms at close distances, M_L (SMA), is slightly biased with respect to the M_L obtained from standard Wood-Anderson seismograms necessarily located at much greater distances (hundreds of kilometers), M_L (WA). For $M_L > 5.3$ the *Luco* [1982] correction is

$$M_L(\text{SMA}) = 0.7 M_L(\text{WA}) + 1.5 \tag{4}$$

and this correction to our model calculations is given by the dashed line in Figure 2.

Observationally, Figure 2 does not leave much to the imagination. There is good reason to believe that earthquakes significantly larger than the 1906 earthquake cannot occur in California [*Hanks and Kanamori*, 1979]. Figure 2, however, also contains data for the smallest earthquakes ($M_L \simeq 0$) that can be recorded in California, at least under ordinary conditions of recording earthquakes at ordinary crustal depths. While there is yet a data gap for $10^{25} \leq M_0 \leq 10^{27}$ dyne cm in central California (Figure 2), we are confident that there are no latent surprises here, if available southern California data for this M_0 range and our model calculations mean anything at all. The model calculations reproduce the continuous log $M_0 - M_L$ curvature in Figure 2 very well (or, if one prefers, c-values that increase with M_L), but a more fundamental result is at hand: consistent with the findings of *Hanks and McGuire* [1981] and *Boore* [1983]—but here expressed for the entire range of earthquakes that can be recorded locally in

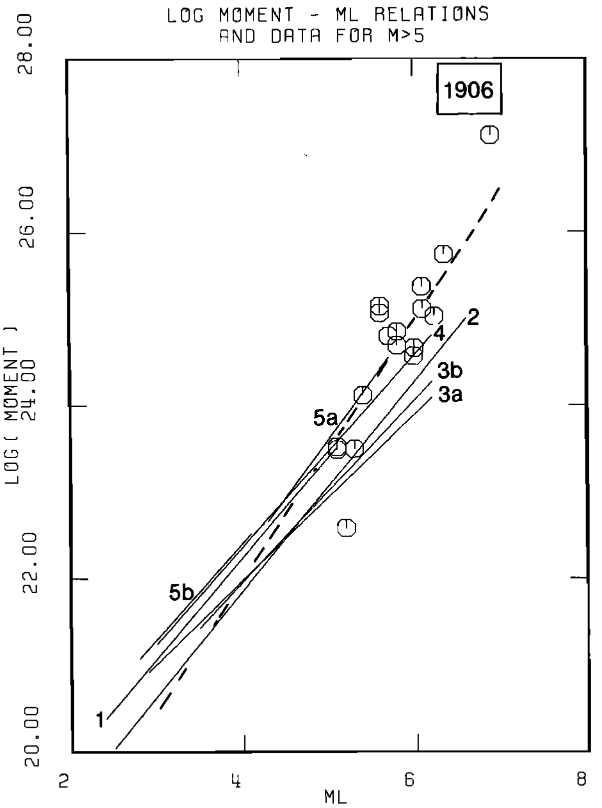

Fig. 1. Moment-magnitude relations and data for 18 central California earthquakes, $M_L \geq 5$. The moment-magnitude relations are given in Table 1: 1, *Johnson and McEvilly* [1974]; 2, *Bakun and Lindh* [1977]; 3, *Archuleta et al.* [1982], *a*, $2.9 \leq M_L \leq 6.2$, *b*, $3.5 \leq M_L \leq 6.2$; 4, *Bolt and Herraiz* [1983]; 5, *Fletcher et al.* [1984], *a*, $4.3 \leq M_L \leq 5.7$, *b*, $2.8 \leq M_L \leq 4.1$. The dashed line is the relation of *Thatcher and Hanks* [1973]. The $M_L \geq 5$ data are from Table 2, the box representing the range of estimates for the 1906 earthquake.

California—the a_{rms} stress drop of 100 bars is a stable (to a factor of 2 or so) and entire feature of California earthquakes.

Asymptotic Approximations to the Numerical Calculations

The curvature of the synthetic moment-magnitude relation in Figure 2 results from a complex interaction in the frequency domain, resulting in three essential bandwidths determined by f_s, the natural frequency of the standard Wood-Anderson torsion seismograph (1.25 Hz); f_{max} (15 Hz); and f_0, the earthquake corner frequency fixed by the constant stress drop relation

$$\Delta\sigma = 100 \text{ bars} = 8.47 \frac{M_0 f_0^3}{\beta^3} \qquad (5)$$

where β is the shear-wave velocity [*Hanks and McGuire*, 1981]. Straight-line approximations of the Wood-Anderson instrumental response at gain V to acceleration and the "ω square" source acceleration spectrum [*Aki*, 1967; *Brune*, 1970] with the f_{max} cutoff [*Hanks*, 1982] are shown at the top of Figure 3. The Wood-Anderson record spectrum is formed from the product of the two, which reduces to addition in the logarithmic space of Figure 3. In passing from the largest earthquakes to the smallest, f_0 sweeps from a value $\ll f_s$ to a value $> f_{max}$. This results in three essential bandwidths for the record spectrum, approximated by the three sketches in the lower part of Figure 3. For each of these, we can extract a linear relation between log M_0 and M_L that approximates the curvature of the calculations (and the observations) with connected straight-line segments, in the following manner.

For each of these boxcar-like spectrums, the maximum Wood-Anderson record amplitude can be estimated with the relation

$$u_{WA} \sim A \Delta f_b \qquad (6)$$

where A is the constant value of spectral amplitudes across the bandwidth Δf_b (Figure 3). For the three cases in Figure 3, Δf_b and u_{WA} are

$$\text{(i)} \quad f_0 \ll f_s \ll f_{max}\colon \Delta f_b = f_s - f_0 \simeq f_s \qquad (7a)$$
$$u_{WA} \sim V M_0 f_0^2 / f_s$$
$$\text{(ii)} \quad f_s \ll f_0 < f_{max}\colon \Delta f_b = f_0 - f_s \simeq f_0 \qquad (7b)$$
$$u_{WA} \sim V M_0 f_0$$
$$\text{(iii)} \quad f_s \ll f_{max} < f_0\colon \Delta f_b = f_{max} - f_s \simeq f_{max} \qquad (7c)$$
$$u_{WA} \sim V M_0 f_{max}$$

Taking $M_L \sim \log u_{WA}$ and $f_0 \sim M_0^{-1/3}$ for constant stress drop (e.g., equation (5)), we can write equations (7) as

$$\text{(i)} \quad \log M_0 \sim 3.0\, M_L \qquad f_0 \ll f_s \qquad (8a)$$
$$\text{(ii)} \quad \log M_0 \sim 1.5\, M_L \qquad f_s \ll f_0 < f_{max} \qquad (8b)$$
$$\text{(iii)} \quad \log M_0 \sim 1.0\, M_L \qquad f_0 > f_{max} \qquad (8c)$$

In view of the highly idealized assumptions leading to equation (8), it would be unwise to make too much of these asymptotic, linear relations. Indeed, our principal interest in them is to illustrate, in a qualitative sense, the nature of the intrinsically complicated model calculations in Figure 2. Even so, the approximations (8) agree with the observational and model results of Figure 2 reasonably well, as described in the paragraphs below.

We estimate with equations (5) and (2) that f_0 should begin to exceed f_{max} at $M_L \simeq 2\frac{1}{2}$. At this magnitude and smaller, then, we infer a c value of 1 (equation (8*c*)), and this seems to be appropriate (e.g., Figure 2 and *Bakun* [1984]). The one-to-one correspondence between M_L and log M_0 when $f_0 \gtrsim f_{max} \gg f_s$ arises because the frequency dependence of u_{WA} is due to f_{max} alone, a fixed parameter (equation (7*c*)). It is incorrect, although commonly held [*Randall*, 1973; *Archuleta et al.*, 1982], that the maximum Wood-Anderson displacement amplitude to a Brune pulse is linearly proportional to M_0 alone when $f_0 > f_s$, yielding a one-to-one relationship between log M_0 and M_L for small events. In fact, in the absence of the effect of f_{max} (i.e., $f_{max} \gg f_0$), u_{WA} of a Brune pulse is proportional to the product $M_0 f_0$ when $f_0 > f_s$. We have captured this result in equation (7*b*), although it may be obtained directly by evaluating the *Brune* [1970, 1971] displacement pulse at the time of maximum displacement. Equations (7*c*) and (8*c*) moreover tell us that when $f_0 > f_{max}$, the M_L dependence on M_0 is insensitive to stress drop, so that our earlier conclusion concerning the ubiquity of the a_{rms} stress drop of 100 bars is only determined at $M_L \gtrsim 2\frac{1}{2}$. Until such time as we know the underlying physical causes of f_{max}, there is no way, in fact, of knowing anything about earthquake stress differences when f_0 exceeds f_{max}.

For $f_0 \simeq 1.25$ Hz ($= f_s$) at $M_L \simeq 4.5$, as suggested by the Oroville aftershocks [*Fletcher et al.*, 1984], the approximations suggest the c value should increase from 1.5 to 3 at

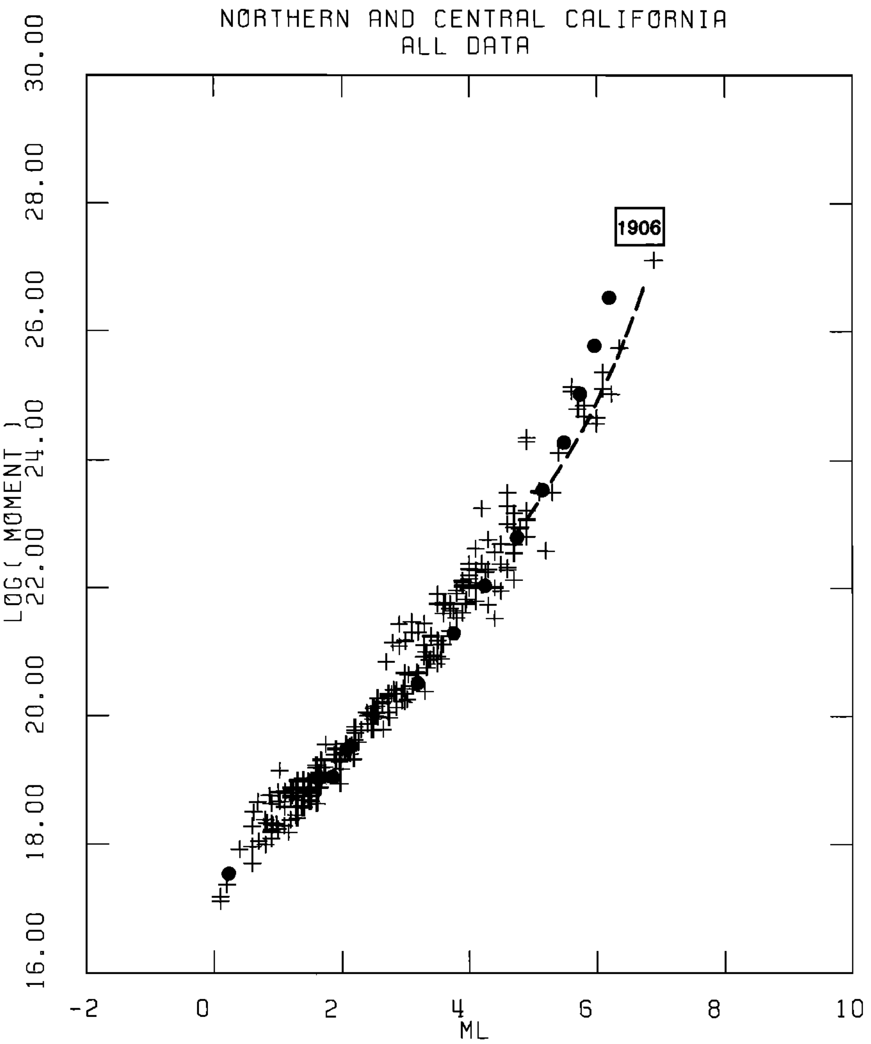

Fig. 2. Moment-magnitude data for central California earthquakes (crosses, box for the 1906 earthquake) and model calculations after *Boore* [1983] (solid circles, dashed line for *Luco* [1982] correction (equation (4)). Data sources: D. J. Andrews (personal communication, 1983), *Archuleta et al.* [1982], *Bakun* [1984], *Bakun and Lindh* [1977], *Fletcher et al.* [1984], *Followill et al.* [1982], *Helmberger and Johnson* [1977], *Helmberger and Malone* [1975], *Uhrhammer* [1981], and Table 2. The model calculations are described in the text.

$M_L \simeq 4\frac{1}{2}$. This asymptotic prediction is less accurate: the observations suggest a steepening above $c = 1.5$ does not occur until $M_L \gtrsim 6$. Above $M_L \simeq 6$, it is hard to define a c value with much accuracy, although $c = 3$ is certainly not inappropriate for $6 \lesssim M_L \lesssim 7$. In the vicinity of $M_L \simeq 7$, the "saturation point" of M_L [*Hanks and Kanamori*, 1979], we suspect that c can obtain arbitrarily large values for reasons not included in the derivation of equations (8): an arbitrarily large earthquake cannot be observed isotropically at any finite distance—any maximum record amplitude will always be associated with some smaller segment of faulting.

SUMMARY AND CONCLUSIONS

The difference in c values for the moment-magnitude relations of central and southern California (Table 1) is a geographic appearance, not a geographic reality; it results from the preponderance of small ($M_L < 5$) earthquakes that form the bulk of the central California data set. The continuous, positive curvature of the log $M_0 - M_L$ observations can be approximated by the linear relations of *Bakun* [1984], reproduced here as equation (3), for $M_L \lesssim 6$. The approprite linear approximation above $M \simeq 6$ has not been defined, nor will it ever be well-defined empirically, if we correctly anticipate that c will become very large in the neighborhood of $M_L = 7$. Neither can it be expected that our asymptotic approximation (8) will be very helpful in this range.

Our full numerical calculations, however, match the continuous curvature of log $M_0 - M_L$ data very well, a data set that represents the entire range of earthquakes that can be locally recorded in California. In view of this fit (Figure 2), we simply need not be concerned about linear approximations: the calculations of *Boore* [1983] allow one to calculate M_L for any M_0, given chosen values of $\Delta\sigma$ and f_{max}. The remarkably good fit of model to data in Figure 2 must mean that the a_{rms} stress drop of 100 bars is a well-conditioned and pervasive property of California earthquakes in the "visible" bandwidth, $f_0 \leq f_{max} \simeq 15$ Hz, corresponding to $M_L \gtrsim 2\frac{1}{2}$.

Finally, it is worth emphasizing that just as the results in Figure 2 (either theoretical or observational) do not depend on the adjectives "central" or "southern" when speaking of California earthquakes, neither do they depend on the modifier "California" when speaking of "plate margin" earthquakes. In a summary of average source-parameter relations for plate

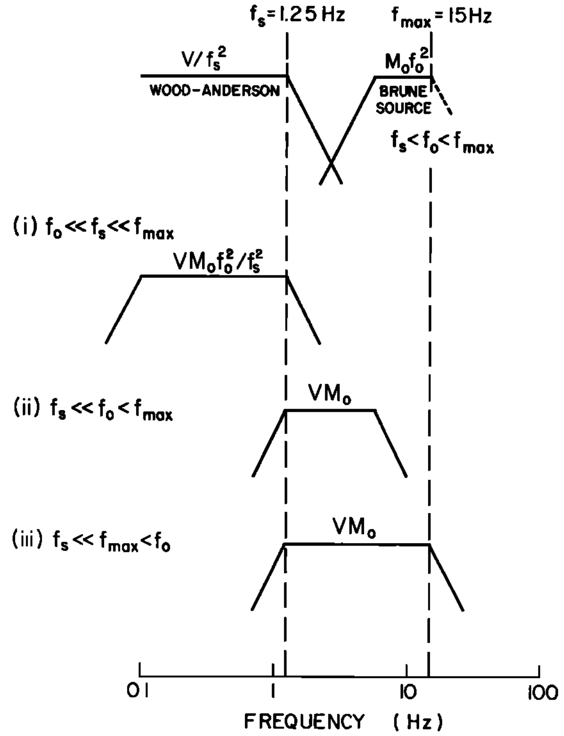

Fig. 3. The Wood-Anderson record spectrum for the three cases (i) $f_0 \ll f_s \ll f_{max}$, (ii) $f_s \ll f_0 < f_{max}$, and (iii) $f_s \ll f_{max} < f_0$. Here we have fixed f_s and f_{max} at 1.25 and 15 Hz, respectively, although the analysis is general for any $f_{max} > f_s$. These three cases, lower part of the figure, are formed from the logarithmic addition of the two idealized spectra in the upper part of the figure. Upper left, Wood-Anderson (at gain V) acceleration response spectrum. Upper right, the Brune source acceleration spectrum in the presence of f_{max}, when $f_s < f_0 < f_{max}$. When $f_{max} < f_0$, the Brune source acceleration spectrum is an asymmetric triangle peaked at f_{max} (not shown).

margin earthquakes on a world-wide basis, *Nuttli* [1984] has determined the relations.

$$\log M_0 = 1.0\, m_b + 18.15 \qquad m_b \leq 4.4 \qquad (9a)$$

$$\log M_0 = 2.0\, m_b + 13.75 \qquad 4.4 \leq m_b \leq 6.9 \qquad (9b)$$

With an origin shift of

$$m_b = M_L - 0.4 \qquad (9c)$$

(O. Nuttli, personal communication, 1984), equations (9*a*) and (9*b*) also fit the observations in Figure 2 well and are very nearly concordant with the model calculations presented there.

Acknowledgments. We have enjoyed the critical commentary of W. H. Bakun, R. B. Herrmann, and A. McGarr in preparing this manuscript for publication. O. Nuttli provided us with a preprint of *Nuttli* [1984] and his thoughts on this study that allowed us to write the paragraph just above at a late date.

REFERENCES

Aki, K., Scaling law of seismic spectrum, *J. Geophys. Res.*, *72*, 1217–1231, 1967.

Archuleta, R. J., E. Cranswick, C. Mueller, and P. Spudich, Source parameters of the 1980 Mammoth Lakes, California, earthquake sequence, *J. Geophys. Res.*, *87*, 4595–4607, 1982.

Bakun, W. H., Seismic moments, local magnitudes, and coda-duration magnitudes for earthquakes in central California, *Bull. Seismol. Soc. Am.*, in press, 1984.

Bakun, W. H., and A. G. Lindh, Local magnitudes, seismic moments, and coda durations for earthquakes near Oroville, California, *Bull. Seismol. Soc. Am.*, *67*, 615–629, 1977.

Bakun, W. H., and T. V. McEvilly, Recurrence models and Parkfield, California, earthquakes, *J. Geophys. Res.*, *89*, in press, 1984.

Barker, J. S., and C. A. Langston, A teleseismic body-wave analysis of the May 1980 Mammoth Lakes, California, earthquakes, *Bull. Seismol. Soc. Am.*, *73*, 419–434, 1983.

Ben-Menahem, A., Source mechanism of the 1906 San Francisco earthquake, *Phys. Earth Planet. Inter.*, *17*, 163–181, 1978.

Bolt, B. A., and M. Herraiz, Simplified estimation of seismic moment from seismograms, *Bull. Seismol. Soc. Am.*, *73*, 735–748, 1983.

Bolt, B. A., T. V. McEvilly, and R. A. Uhrhammer, The Livermore Valley, California, sequence of January 1980, *Bull. Seismol. Soc. Am.*, *71*, 451–463, 1981.

Boore, D. M., Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra, *Bull. Seismol. Soc. Am.*, *73*, 1865–1894, 1983.

Boore, D. M., Use of seismoscope records to determine M_L and peak velocities, *Bull. Seismol. Soc. Am.*, *74*, 315–324, 1984.

Brune, J. N., Tectonic stress and the spectra of seismic shear waves from earthquakes, *J. Geophys. Res.*, *75*, 4997–5009, 1970.

Brune, J. N., Correction, *J. Geophys. Res.*, *76*, 5002, 1971.

Brune, J. N., A. Espinosa, and J. Oliver, Relative excitation of surface waves by earthquakes and underground explosions in the California-Nevada region, *J. Geophys. Res.*, *68*, 3501–3513, 1963.

Burdick, L. J., Broad-band seismic studies of body waves, Ph.D. thesis, 151 pp., Calif. Inst. Technol., Pasadena, 1977.

Cohn, S. N., T. Hong, and D. V. Helmberger, The Oroville earthquakes: A study of source characteristics and site effects, *J. Geophys. Res.*, *87*, 4585–4594, 1982.

Ferguson, R., B. Schechter, and R. A. Uhrhammer, Bulletin of the Seismographic Stations of the University of California, vol. 50, number 1, 87 pp., Univ. of Calif., Berkeley, 1980.

Fletcher, J. B., J. Boatwright, L. C. Haar, T. C. Hanks, and A. McGarr, Source parameters for aftershocks of the Oroville, California, earthquake, *Bull. Seismol. Soc. Am.*, in press, 1984.

Followill, F. E., J. M. Mills, Jr., and R. F. Rohrer, A comparison of magnitudes and moments for 26 Livermore Valley microearthquakes, *Bull. Seismol. Soc. Am.*, *72*, 1025–1031, 1982.

Given, J. W., T. C. Wallace, and H. Kanamori, Teleseismic analysis of the 1980 Mammoth Lakes earthquake sequence, *Bull. Seismol. Soc. Am.*, *72*, 1093–1109, 1982.

Hanks, T. C., f_{max}, *Bull. Seismol. Soc. Am.*, *72*, 1867–1879, 1982.

Hanks, T. C., and H. Kanamori, A moment magnitude scale, *J. Geophys. Res.*, *84*, 2348–2350, 1979.

Hanks, T. C., and R. K. McGuire, The character of high frequency strong ground motion, *Bull. Seismol. Soc. Am.*, *71*, 2071–2095, 1981.

Helmberger, D. V., and L. R. Johnson, Source parameters of moderate size earthquakes and the importance of receiver crustal structure in interpreting observations of local earthquakes, *Bull. Seismol. Soc. Am.*, *67*, 301–313, 1977.

Helmberger, D. V., and S. D. Malone, Modeling local earthquakes as shear dislocations in a layered halfspace, *J. Geophys. Res.*, *80*, 4881–4888, 1975.

Jennings, P. C., and H. Kanamori, Determination of local magnitude, M_L, from seismoscope records, *Bull. Seismol. Soc. Am.*, *69*, 1267–1288, 1979.

Johnson, L. R., and T. V. McEvilly, Near-field observations and source parameters of central California earthquakes, *Bull. Seismol. Soc. Am.*, *64*, 1855–1886, 1974.

Kanamori, H., The energy release in great earthquakes, *J. Geophys. Res.*, *82*, 2981–2987, 1977.

Kanamori, H., and P. C. Jennings, Determination of local magnitude, M_L, from strong motion accelerograms, *Bull. Seismol. Soc. Am.*, *68*, 471–485, 1978.

Langston, C. A., and R. Butler, Focal mechanism of the August 1, 1975 Oroville earthquake, *Bull. Seismol. Soc. Am.*, *66*, 1111–1120, 1976.

Lay, T., J. W. Given, and H. Kanamori, Long-period mechanism of the 8 November 1980 Eureka, California, earthquake, *Bull. Seismol. Soc. Am.*, *72*, 439–456, 1982.

Liu, H., and D. V. Helmberger, The near-source ground motion of the 6 August 1979 Coyote Lake, California, earthquake, *Bull. Seismol. Soc. Am.*, *73*, 201–218, 1983.

Luco, J. E., A note on near-source estimates of local magnitude, *Bull. Seismol. Soc. Am.*, *72*, 941–958, 1982.

McKenzie, M. R., R. D. Miller, and R. A. Uhrhammer, Bulletin of the

Seismographic Stations of the University of California, vol. 50, number 2, 156 pp., Univ. of Calif., Berkeley, 1980.
Morrison, P. W., Jr., B. W. Stump, and R. Uhrhammer, The Oroville earthquake sequence of August 1975, *Bull. Seismol. Soc. Am.*, *66*, 1065–1084, 1976.
Nuttli, O., Average seismic source-parameter relations for plate-margin earthquakes, *Tectonophysics*, in press, 1984.
Purcaru, G., and H. Berckhemer, A magnitude scale for very large earthquakes, *Tectonophysics*, *49*, 189–198, 1978.
Randall, M. J., The spectral theory of seismic sources, *Bull. Seismol. Soc. Am.*, *63*, 1133–1144, 1973.
Ryall, A., J. D. VanWormer, and A. E. Jones, Triggering of microearthquakes by earth tides, and other features of the Truckee, California, earthquake sequence of September, 1966, *Bull. Seismol. Soc. Am.*, *58*, 215–248, 1968.
Thatcher, W., Strain accumulation and release mechanism of the 1906 San Francisco earthquake, *J. Geophys. Res.*, *80*, 4862–4872, 1975.
Thatcher, W., and T. C. Hanks, Source parameters of southern California earthquakes, *J. Geophys. Res.*, *78*, 8547–8576, 1973.
Tsai, Y., and K. Aki, Simultaneous determination of the seismic moment and attenuation of seismic surface waves, *Bull. Seismol. Soc. Am.*, *59*, 275–287, 1969.
Tsai, Y., and K. Aki, Source mechanism of the Truckee, California, earthquake of September 12, 1966, *Bull. Seismol. Soc. Am.*, *60*, 1199–1208, 1970.
Uhrhammer, R. A., Observations of the Coyote Lake, California earthquake sequence of August 6, 1979, *Bull. Seismol. Soc. Am.*, *70*, 559–570, 1980.
Uhrhammer, R. A., The Pacifica earthquake of 28 April 1979, *Bull. Seismol. Soc. Am.*, *71*, 1161–1172, 1981.
Uhrhammer, R. A., and R. W. Ferguson, The 1980 Mammoth Lakes earthquake sequence, *Spec. Rep. Calif. Div. Mines Geol.*, *150*, 131–136, 1980.
Wyss, M., and J. N. Brune, Seismic moment, stress, and source dimensions for earthquakes in the California-Nevada region, *J. Geophys. Res.*, *73*, 4681–4694, 1968.

D. M. Boore and T. C. Hanks, U.S. Geological Survey, 345 Middlefield Road, Menlo Park, CA 94025.

(Received August 22, 1983;
revised January 27, 1984;
accepted March 13, 1984.)