# NOTES ON THE EQUATION TO USE FOR PGA4NL 

By

David M. Boore<br>Gail M. Atkinson

Our ground-motion predictions equations (GMPEs) (Boore and Atkinson, 2007 and 2008) (BA07 and BA08), include a non-linear site amplification term that requires a value of peak acceleration (PGA)) for the magnitude and distance for which the equations are being evaluated. We call this quantity pga4nl. We obtained an equation for pga4nl during our initial regression work, as described in BA07. That equation is given by the regression coefficients in the first row of Tables 4.2 and 4.4 in BA07. The observed data were modified to a reference condition of $V_{S 30}=760 \mathrm{~m} / \mathrm{s}$, using the initial regression equation for pga4nl, and the final ground-motion prediction equations (GMPEs) were developed using this modified set of observed data. The peak accelerations for the reference site condition from the final regression coefficients (the second row in Tables 6 and 7 in BA08) are not the same as given by the coefficients from the initial regression. This has the unfortunate consequence that a plot of site amplification against peak acceleration developed from our GMPEs will show a dependence on magnitude, distance, and fault type, if the points used in the plot are developed in the following reasonable way (for a given oscillator period):

1. For a range of magnitudes and distance, and a given fault type, evaluate $P S A$ for a specified $V_{S 30}$ and for the reference condition of $V_{530}=760 \mathrm{~m} / \mathrm{s}$, using the BA07 equation for $p g a 4 n l$.
2. Calculate the site amplification as $P S A\left(V_{S 30}\right) / P S A(760)$.
3. Compute $P G A(760)$, using the final GMPEs
4. Plot the site amplification against the PGA for each magnitude and distance, for the specified fault type.

Figure 1 shows the site amplification computed as stated above for $V_{\text {S30 }}=180 \mathrm{~m} / \mathrm{s}$ and $T=0.2 \mathrm{~s}$. A range of magnitudes from 5 to 8 in 0.5 magnitude increments and distances from 0.1 to 200 km were used. Site amplifications were computed for normal and for strikeslip faults.


Figure 1. Amplification vs. peak acceleration, using the coefficients in the first row of Tables 6 and 7 for pga4nl. Each colored symbol represents the motions computed for a particular magnitude, distance, and fault type.

The amplifications are multivalued, depending on magnitude, distance, and fault type, which is an unintended result. We think at the level of sophistication for site response in our paper that site amplification should only be a function of the peak acceleration for the reference site condition. The multivalued nature of the site amplification is a direct consequence of the inconsistency of $P G A$ from the initial and the final regressions. This inconsistency is shown in Figure 2.


Figure 2. Peak acceleration against distance for the reference site condition ( $V_{S 30}=760 \mathrm{~m} / \mathrm{s}$ ) for strikeslip and normal fault types, computed using the two alternate equations for pga4nl (row 1 in Tables 4.2 and 4.4 in BA07 and row 2 in Tables 6 and 7 in BA08). The initial equations did not allow for a magnitude-dependent geometrical spreading term, which explains some of the differences between the blue and the red curves. The similarity of the curves from the initial regression for both magnitudes is a result of the magnitude saturation in the equations.

It is clear from the above figure that a given value of $P G A$ from the initial regression will correspond to several values of PGA from the final GMPEs. The scatter in Figure 1 results from plotting the amplification against the $P G A$ from the final regression rather than the PGA used in evaluating the site amplification. If, on the other hand, we specify that pga4nl is to be obtained by evaluating our final GMPEs (using the coefficients in the second row of Tables 6 and 7 in BA08), the plot of amplifications against $P G A$ is now single valued, as shown in Figure 3.


Figure 3. Amplification vs. peak acceleration, using the coefficients in the second row of Tables 6 and 7 of BA08 for pga4nl. The black curves are from a direct evaluation of the subroutine used to calculate site response (which depends only on PGA and not on fault type, magnitude, or distance); the agreement with the amplifications obtained using the more round-about procedure outline above is a consistency check for our evaluation program.

What is the consequence for ground motions from our equations of using these alternate equations for pga4nl? We show in Figures 4, 5, and 6 ground motions using the two procedures for the determination of pga4nl. Figure 4, showing PSA at two periods for $\mathbf{M}=7$, is a modification of Figure 12 in BA08. It shows little difference in the curves. As expected from Figure 2, however, the difference in predicted motions becomes more important as magnitude increases, for distances between about 10 and 100 km (for great enough distance, there is no nonlinear response, so the value of pga4nl is immaterial).


Figure 4. PSA vs. distance for $\mathbf{M} 7.0$, for 0.2 and 3 s oscillators. This is a modification of Figure 12 in BA08..


Figure 5. PSA vs. distance for $\mathbf{M} 7.5$, for 0.2 and 3 s oscillators.


Figure 6. PSA vs. distance for $\mathbf{M} 8.0$, for 0.2 and 3 s oscillators.

Overall, these are minor issues. They could have been avoided if we had iterated, using the coefficients in the second row of Tables 6 and 7 in BA08 to do a new modification of the observed data to the reference site condition, and then using the newly modified data to obtain new regression coefficients (and so on). We are not convinced this additional step is warranted as its main impact would be a minor improvement in internal consistency. However, to avoid such inconsistency we recommend that the final PGA regression equation in BA08 be used to estimate pga4nl, instead of the initial version we provided in BA07. Using the final GMPEs for pga4nl will avoid the scatter seen in Figure 1. Differences in predicted ground motions are small for most conditions. We would not think it is necessary to repeat any analyses that may have already been done using pga4nl as initially recommended (due to the minor nature of the change).

## References

Boore, D. M. and Atkinson, G. M., 2007. Boore-Atkinson NGA Ground Motion Relations for the Geometric Mean Horizontal Component of Peak and Spectral Ground Motion Parameters, PEER Report 2007/01, Pacific Earthquake Engineering Center, Berkeley, California

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