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ON LOW-FREQUENCY ERRORS OF UNIFORMLY MODULATED FILTERED WHITE-NOISE MODELS FOR GROUND MOTIONS

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SUMMARY

Low-frequency errors of a commonly used non-stationary stochastic model (uniformly modulated filtered white-noise model) for earthquake ground motions are investigated. It is shown both analytically and by numerical simulation that uniformly modulated filter white-noise-type models systematically overestimate the spectral response for periods longer than the effective duration of the earthquake, because of the built-in low-frequency errors in the model. The errors, which are significant for low-magnitude short-duration earthquakes, can be eliminated by using the filtered shot-noise-type models (i.e. white noise, modulated by the envelope first, and then filtered).

INTRODUCTION

Stochastic methods for ground motion description and seismic structural response estimation are gaining acceptance as a means of specifying design motions. The random nature of earthquakes and the increasing amount of recorded ground motion data make it more attractive to use a probabilistic approach rather than a deterministic approach to solve earthquake related problems.

A standard way of describing ground motion in stochastic terms for structural response calculations has been to use a filtered white noise multiplied by a deterministic envelope function. An approximation in this method, usually unstated, is that multiplying by the envelope function does not affect the frequency content of the signal. The approximation is based on the assumption that the time variation of the envelope function is much slower than that of the filtered signal.

However, as will be shown here, this approximation introduces a built-in low-frequency error in the model. The error is significant for structures whose natural periods are greater than the source duration of the earthquake. The error in the model can be eliminated if the order of filtering and multiplying by the envelope of the white noise is interchanged. This corresponds to the so-called filtered shot-noise model.

This study will first show analytically that the filtered, then enveloped, white-noise model has low-frequency errors, and that the errors can be eliminated by using the filtered shot-noise model. Then, by using simulated time series, the errors in the models and their effect on response spectra will be investigated for three different earthquakes. The simulations are based on seismological models of strong ground motions and can create acceleration time series for any earthquake with specified source and attenuation properties.

STOCHASTIC MODELS FOR EARTHQUAKE GROUND MOTION

A stochastic model that has been used extensively in the past to describe strong ground motions is a filtered white noise whose amplitude is modulated by a deterministic envelope function. For brevity, this model will be denoted as UMFWN (uniformly modulated filtered white noise) for the remainder of the paper. Analytically,

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the UMFWN model is described by the following equation for the ground acceleration, $a(t)$:

$$a(t) = w(t)x(t) \tag{1}$$

where $w(t)$ is the deterministic envelope function (time window) and $x(t)$ is a zero mean stationary random process. The major assumption of the model is that $w(t)$ varies much more slowly with time than $x(t)$, such that multiplication by $w(t)$ does not change the frequency content of $x(t)$. The model given by equation (1) has been used by a large number of authors for seismic response studies (see Reference 3 for an extensive review).

For a typical envelope function $w(t)$, the frequency content of $a(t)$ cannot be made equal to that of $x(t)$ for all frequencies. This can be shown by calculating the Fourier amplitude spectrum of $a(t)$. The amplitude spectrum $A_a(f)$ of $a(t)$ is equal to the amplitude of the convolution of the Fourier transform $F_w(\mathrm{i}f)$ of $w(t)$, and the Fourier transform $F_x(\mathrm{i}f)$ of $x(t)$, given by

$$A_a(f) = 2\pi \left| \int_{-\infty}^{\infty} F_w(\mathrm{i}f_1) F_x(\mathrm{i}f - \mathrm{i}f_1)\,\mathrm{d}f_1 \right| \tag{2}$$

The assumption that $A_a(f) = A_x(f)$ would be valid only if $|F_w(\mathrm{i}f)| = \delta(f)$, where $\delta(f)$ is the Delta function. For a limited-duration envelope function $|F_w(\mathrm{i}f)|$ is not a Delta function, although it peaks at zero frequency. Moreover, from physical reasons it is required that $A_a(0) = 0$. From equation (2), however, $A_a(0) \neq 0$, even though $A_x(0) = 0$. In terms of loads on structures, having a non-zero value for $A_a(0)$ corresponds to a fictitious static load, which does not exist in reality. The magnitude of errors involved in modelling and in calculated response will be shown later by using simulated accelerograms.

An alternative approach to modelling ground acceleration is to use filtered shot noise (i.e. white noise, first multiplied by an envelope function, then filtered). For brevity, the filtered shot noise will be denoted as FSN for the remainder of the paper. Analytically, the FSN model can be written as

$$a(t) = \int_0^t h_x(t-\tau)s(\tau)\,\mathrm{d}\tau \tag{3}$$

where the shot noise $s(\tau)$ is defined as $s(\tau) = w(\tau)n(\tau)$. $w(\tau)$ is the envelope function, $n(\tau)$ is the white noise, and $h_x(t-\tau)$ is the target ground motion filter in the time domain (i.e. impulse response function of the ground). The time-domain convolution in equation (3) becomes a multiplication in the frequency domain, as

$$A_a(f) = A_x(f)A_s(f) \tag{4}$$

where $A_s(f)$ is the Fourier amplitude spectrum of the shot noise. It is clear from equation (4) that, since by definition $A_x(0) = 0$ and $A_s(0) \neq \infty$, then $A_a(0) = 0$. To prove that $A_a(f) = A_x(f)$ not only at zero frequency but for all the frequencies, it is sufficient to show that shot noise has a constant amplitude spectrum in the mean. The Fourier transform $F_s(\mathrm{i}f)$ of $s(t)$ is

$$F_s(\mathrm{i}f) = \int_0^{\infty} w(t)n(t)\mathrm{e}^{-\mathrm{i}2\pi f t}\,\mathrm{d}t \tag{5}$$

and the amplitude spectrum, $A_s(f)$, is given by

$$A_s^2(f) = F_s(\mathrm{i}f)F_s^*(-\mathrm{i}f) = \int_0^{\infty}\int_0^{\infty} w(t_1)w(t_2)n(t_1)n(t_2)\mathrm{e}^{-\mathrm{i}2\pi(t_1-t_2)f}\,\mathrm{d}t_1\,\mathrm{d}t_2 \tag{6}$$

where * denotes the complex conjugate. The ensemble average $E[A_s^2(f)]$ of $A_s^2(f)$ can be calculated by taking the ensemble average of the right hand side. Since integration is a linear operation, averaging can be done over the integrand, in which the only random variable is $n(t)$. Since $n(t)$ is a white noise,

$$E[n(t_1)n(t_2)] = \sigma_n^2\,\delta(t_1 - t_2) \tag{7}$$

where $\delta(t_1 - t_2) = 1$ for $t_1 = t_2$, and 0 otherwise. Therefore, by putting $t_1 = t_2$ the double integral reduces to a single integral and becomes

$$E[A_s^2(f)] = \sigma_n^2 \int_0^{\infty} w^2(t)\,\mathrm{d}t \tag{8}$$

This expression is independent of f; therefore, it is constant. By scaling the white noise such that

$$\sigma_n^2 = \left[\int_0^\infty w^2(t)\,dt \right]^{-1} \tag{9}$$

$E[A_s(f)]$ can be made equal to unity.

SIMULATED TIME SERIES FOR UMFWN AND FSN MODELS

Although we have demonstrated the low-frequency distortion produced by the UMFWN model, the actual importance of this distortion is its effect on the calculated peak response of a structure. This requires construction of simulated time series that will later be used to investigate the modelling errors and their effect on response spectra. To generate the time series, first the frequency content and the envelope of the motion need to be specified.

The frequency content will be defined by using the seismological model presented in Reference 1. This model gives the Fourier amplitude spectrum of ground acceleration in terms of the physical parameters of the source, attenuation and the site, such as magnitude, distance, wave velocity, stress drop, etc. The envelope function that will be used is of the exponential form, suggested by Shinozuka and Sato.[6] More on the Fourier amplitude spectrum and the envelope used in the simulation can be found in Reference 5.

Once the envelope and Fourier amplitude spectrum of the ground acceleration are specified, a family of corresponding time series can be simulated easily. The steps of simulation for UMFWN and FSN models are as follows.

To generate time histories for the UMFWN model:

(a) generate a white-noise sequence in the time domain, whose spectral amplitude averaged over the frequencies is equal to one;
(b) multiply the spectrum of the white noise with the specified spectrum, and transform back to time domain;
(c) multiply by the envelope function.

To generate time histories for the FSN model:

(a) generate a white-noise sequence in the time domain, and multiply it by the envelope function;
(b) scale the enveloped white noise such that its spectral amplitude averaged over the frequencies is equal to one;
(c) multiply the spectrum of the enveloped white noise with the specified spectrum, and transform back to the time domain.

It should be noted here that there are other methods of generating simulated ground motion time series (see, for example, Reference 2). The above method of simulation is chosen because it matches with the objective of the paper. It should also be noted that the response spectra can be calculated analytically both for UMFWN and FSN models.[4–6] The analytical approach, however, requires some approximations and assumptions on calculating the peak response. Moreover, with the speed of today's computers, numerical simulation can be done as fast as analytical calculations.

NUMERICAL EXAMPLES

In this section, by using the methodology described above, acceleration time series are generated for both UMFWN and FSN models, and the resulting response spectra of a simple oscillator are investigated. Time series are generated for three earthquakes with moment magnitudes 5, 6 and 7, all with a distance of 10 km from the fault. The effective durations of the earthquakes, as defined by Trifunac and Brady,[7] are 10, 3 and 1 s for magnitudes 7, 6 and 5, respectively. It is assumed for all three earthquakes that the effective durations are equal to source durations (i.e. faulting durations) and the corner frequencies can be approximated as the inverse of the effective durations.[5] The time that the envelope reaches its peak is taken 0·20 times the effective duration, and the parameters of the envelope function are calculated accordingly, for each earthquake. The site amplification factor is assumed equal to one, a value representative for rock sites. The values for the remaining parameters of the model are equal to those used in Reference 1.

Twenty acceleration time series are generated for each earthquake. The same seed for the random number generator is used for both UMFWN and FSN models. Figure 1 gives a sample of time series generated by each model for the magnitude 7 earthquake. First, the accuracy of the simulation is checked by comparing the ensemble average of the envelope function and the amplitude spectrum of the simulated series with their target values. This comparison is given for the magnitude 7 earthquake in Figures 2(a–b) for the UMFWN model,

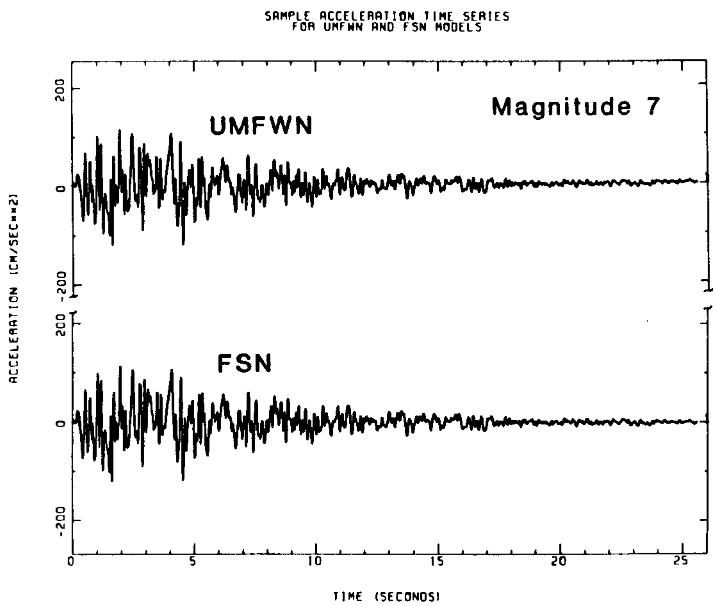

Figure 1. A sample of acceleration time series generated by UMFWN and FSN models for magnitude 7 earthquake

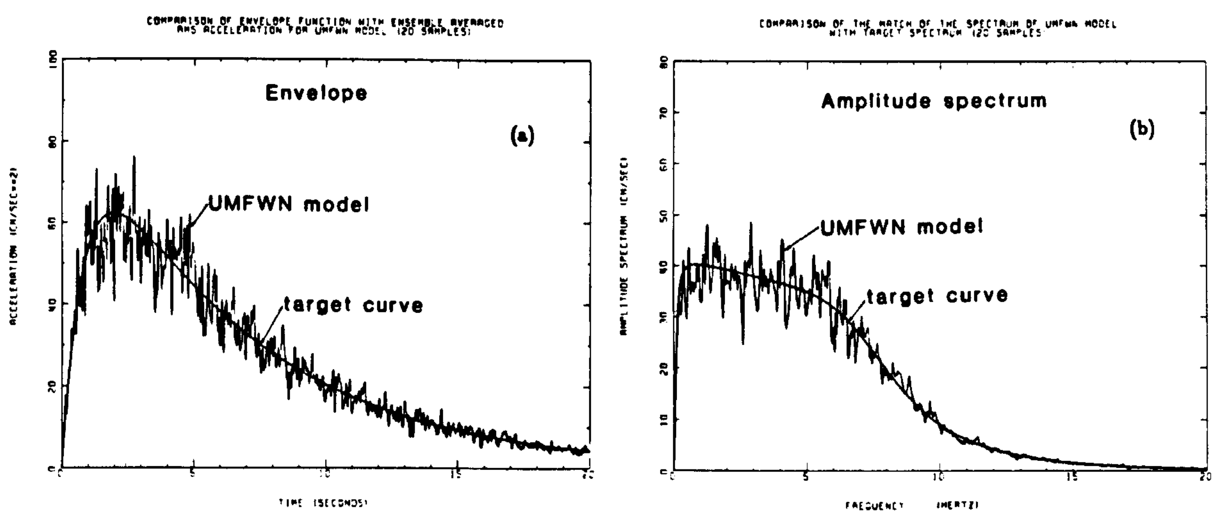

Figure 2. Match of the simulation with target values for magnitude 7 earthquake, UMFWN model, 20 samples: (a) ensemble average of the envelope function; (b) ensemble average of the Fourier amplitude spectra

and in Figures 3(a–b) for the FSN model. As the figures show, the match and the convergence of the simulation are very good even with only 20 samples.

However, as was proved analytically earlier, the UMFWN approach has a built-in low-frequency error. This can be seen by taking a closer look at the low-frequency region of the match for the amplitude spectra. This is presented in Figure 4, which gives the same information given in Figures 2(b) and 3(b) in a larger scale for frequencies zero to twice the corner frequency for the magnitude 7 earthquake. As the figure shows, the amplitude spectrum of the FSN model follows the target curve very well and becomes zero at zero frequency, whereas the amplitude spectrum for the UMFWN model diverges from the target curve and ends at a non-zero value at zero frequency. The divergence starts around the corner frequency (i.e. at $f/f_c = 1$). As the corner

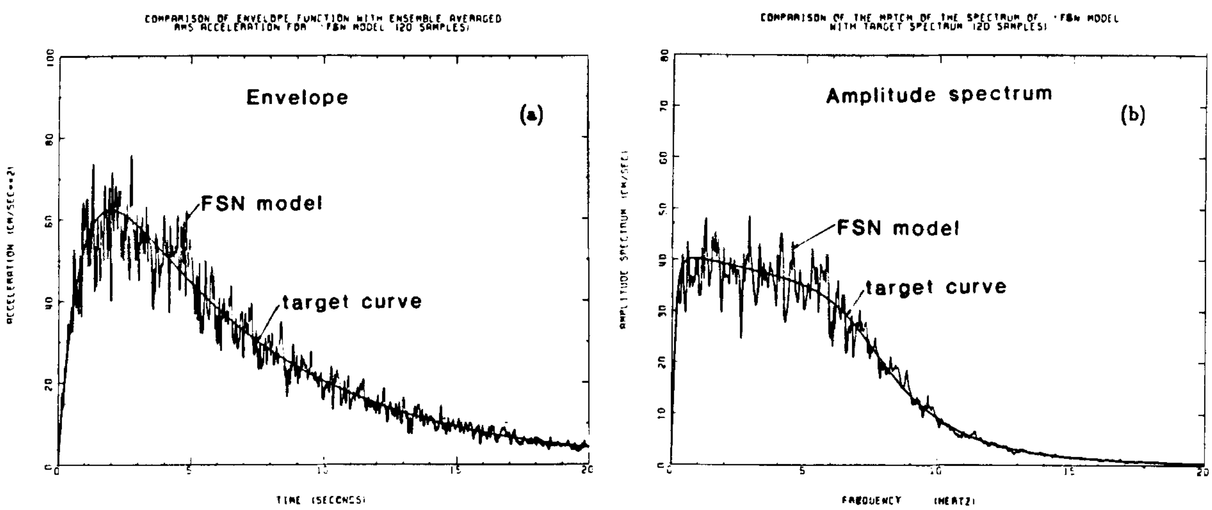

Figure 3. Match of the simulation with target values for magnitude 7 earthquake, FSN model, 20 samples: (a) ensemble average of the envelope function; (b) ensemble average of the Fourier amplitude spectra

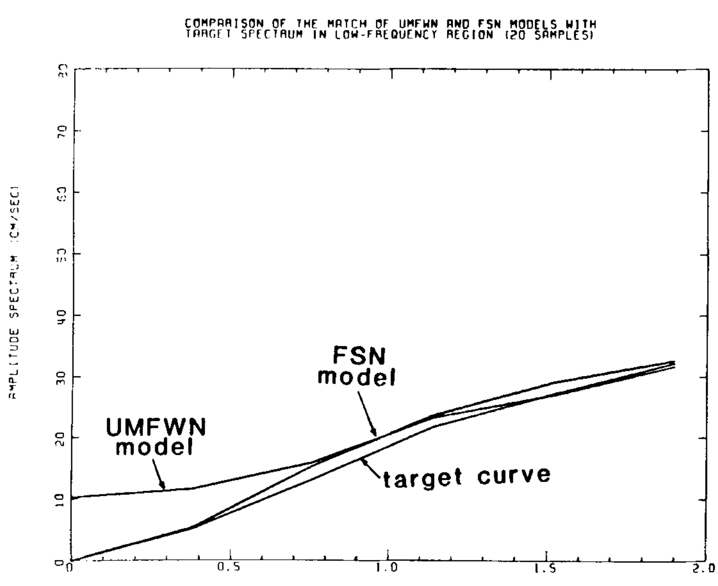

Figure 4. Match of the ensemble average of Fourier amplitude spectra of UMFWN and FSN models with the target curve in the low-frequency region (for frequencies from zero to twice the corner frequency, 20 samples)

frequency becomes higher (i.e. the magnitude and duration becomes smaller), the length of the low-frequency band in which there is mismatch becomes larger. This is the case for low magnitude and short duration earthquakes. The match for the amplitude spectrum of the FSN model is very good, including low frequencies. Similar observations were also made for the match of the magnitude 6 and 5 earthquakes.

Next, by using the simulated time histories, the response of a single degree of freedom oscillator with 5 per cent damping is calculated for each of the 20 sample time histories for each magnitude and model. The results are presented in Figures 5, 6 and 7 for magnitudes 5, 6 and 7, respectively, in terms of maximum, mean and

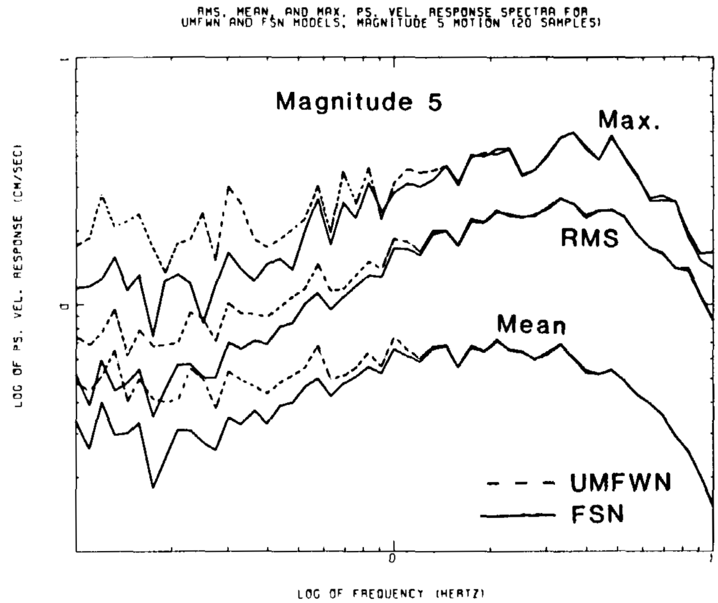

Figure 5. Maximum, mean and RMS pseudo-velocity response spectra calculated from UMFWN and FSN models for magnitude 5 earthquake

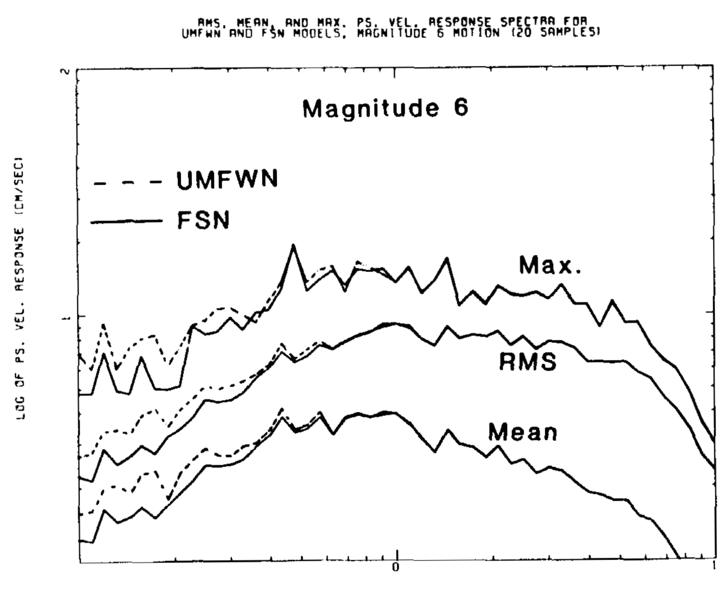

Figure 6. Maximum, mean and RMS pseudo-velocity response spectra calculated from UMFWN and FSN models for magnitude 6 earthquake

RMS pseudo-velocity response spectra for oscillator frequencies 0·1 to 10 Hz. The maximum spectral value at any frequency is the maximum of 20 sample spectral values, whereas the mean spectrum is the arithmetic mean, and the RMS spectrum is the RMS value of 20 values. The figures show that the UMFWN model always overestimates the spectral response for oscillators whose natural frequencies are less than the corner frequency of the ground motion (i.e. for oscillators whose periods are larger than the effective duration of the earthquake, since the corner frequency is approximated as the inverse of the effective duration). This is demonstrated more clearly in Figure 8, which gives the ratio of the mean spectral response calculated by using the UMFWN model

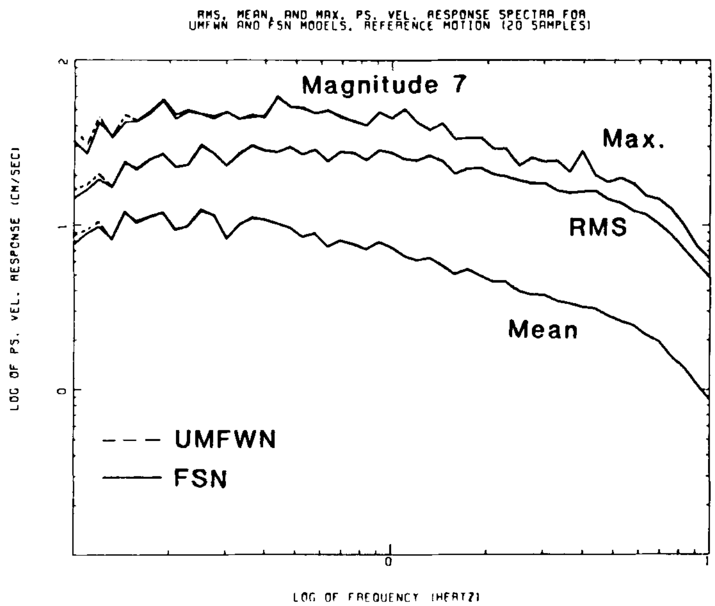

Figure 7. Maximum, mean and RMS pseudo-velocity response spectra calculated from UMFWN and FSN models for magnitude 7 earthquake

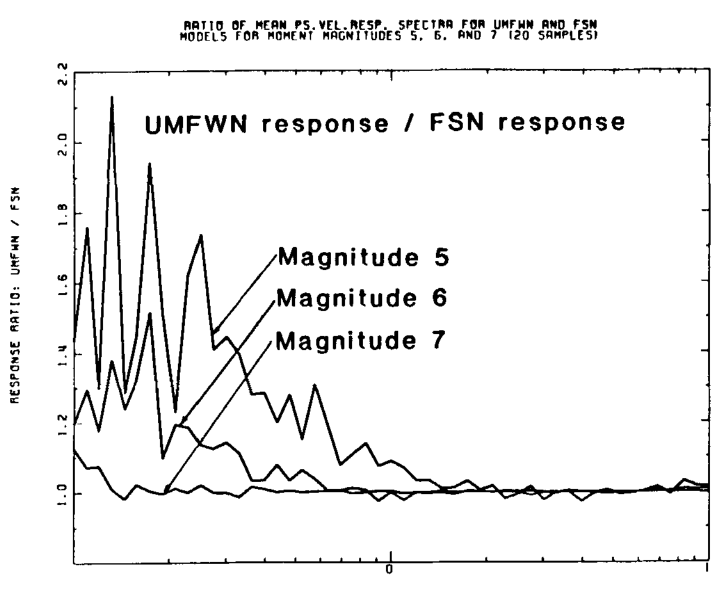

Figure 8. Ratio of pseudo-velocity response spectra due to UMFWN model to that due to FSN model for magnitudes 5, 6 and 7

to that calculated by using the FSN model for each magnitude. The overestimation can be as much as a factor of two. For the remaining frequency band, the two models basically give the same result.

SUMMARY AND CONCLUSIONS

It has been shown both analytically, and by numerical simulation, that a commonly used stochastic model for ground motion description, the uniformly modulated filtered white-noise (UMWFN) model, introduces a systematic error in response spectra for periods larger than the effective duration (i.e. source duration) of the earthquake. For large earthquakes (i.e. moment magnitude 7 and above), the effective duration is generally larger than the period range that is of interest in engineering design; therefore, the error does not have any significance. For smaller and shorter-duration earthquakes, however, the model overestimates the spectral response by as much as a factor of two. We show both analytically and by numerical simulation that the error can be eliminated by using the filtered shot-noise model (i.e. white noise, modulated by the envelope first, and then filtered).

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