A smooth transition between two $(R/R_{REF})^{\gamma}$ geometrical spreading functions

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In an as-yet unpublished report, Peter Stafford uses the following equation for a geometrical spreading function (I'm changing the signs of γ from his equation)

$$\ln g(R) = \gamma_1 \ln(R) + (\gamma_2 - \gamma_1)\ln(\sqrt{R^2 + 50^2}) - (\gamma_2 - \gamma_1)\ln(\sqrt{1^2 + 50^2}) \quad (1)$$

This function approaches $\ln g(R) = \gamma_1 \ln R$ for small *R* and $\ln g(R) = \gamma_2 \ln R$ for large *R*, with a smooth transition at *R*= 50 km between these limits.

Equation (1) can be written more compactly as

$$\ln g(R) = \gamma_1 \ln(R) + (\gamma_2 - \gamma_1)\frac{1}{2}\ln\left(\frac{R^2 + 50^2}{1 + 50^2}\right) \quad (2)$$

To generalize:

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) + (\gamma_2 - \gamma_1)\frac{1}{\xi}\ln\left(\frac{R^{\xi} + R_T^{\xi}}{R_{REF}^{\xi} + R_T^{\xi}}\right) \quad (3)$$

or better (easier to see asymptotic values)

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) + (\gamma_2 - \gamma_1)\frac{1}{\xi}\ln\left(\frac{(R/R_{REF})^{\xi} + (R_T/R_{REF})^{\xi}}{1 + (R_T/R_{REF})^{\xi}}\right) \quad (4)$$

where the rate of transition from γ_1 to γ_2 around the transition distance R_T is controlled by ξ. R_{REF} is the distance at which $g = 1$.

Larger ξ gives a sharper transition (e.g., $\xi = 10$ produces a spreading similar to a bilinear spreading). To make this more apparent, here is an equation for ξ in terms of the ratio $g_{RAT} = g(R_T)/g1(R_T)$, where $g1(R) = \left(R/R_{REF}\right)^{\gamma_1}$:

$$\xi = \left(\gamma_2 - \gamma_1\right)\ln 2/\ln g_{RAT} \tag{5}$$

and

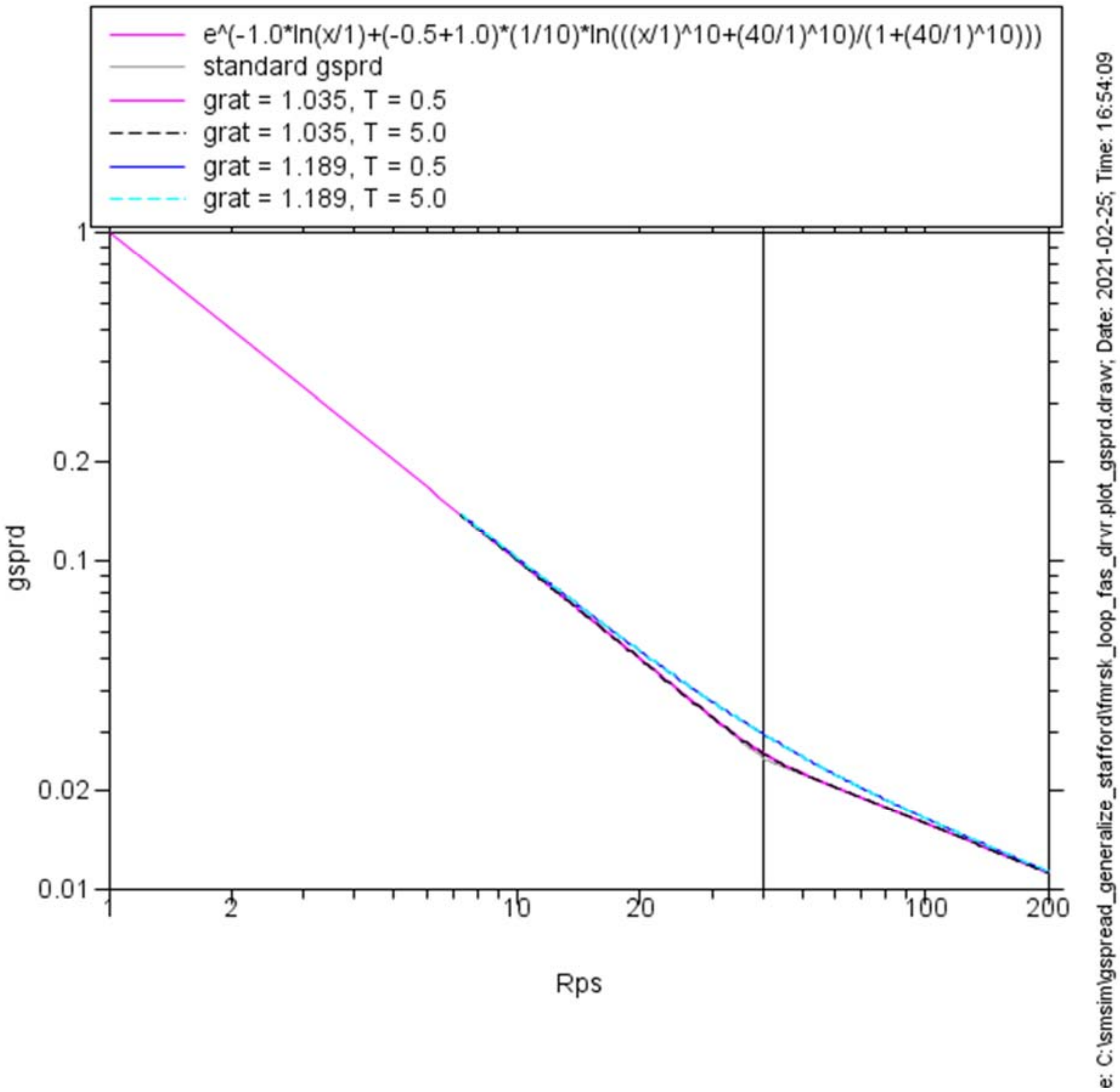
$$g_{RAT} = 2\exp\left(\left(\gamma_2 - \gamma_1\right)/\xi\right). \tag{6}$$

Substituting equation (5) into equation (4) gives:

$$\ln g(R) = \gamma_1 \ln(R/R_{REF}) + \frac{\ln g_{RAT}}{\ln 2}\ln\left(\frac{(R/R_{REF})^{\xi} + (R_T/R_{REF})^{\xi}}{1 + (R_T/R_{REF})^{\xi}}\right) \tag{7}$$

The actual ratio $g(R_T)/g1(R_T)$ resulting from the ξ given by equation (5) asymptotically approaches g_{RAT} for $R_T/R_{REF} >> 1$. But for practical purposes, when R_T is several tens of km and $R_{REF} = 1$, the actual ratio is very close to the asymptotic value. With Peter Stafford's $\xi = 2$, and $\gamma_1 = -1.0$ and $\gamma_2 = -0.5$, equation (6) gives $g_{RAT} = 1.189$. That value is used in the graphs below. In his report, Peter Stafford used $\gamma_1 = -1.158$, $\gamma_2 = -0.5$, and $\xi = 2$, which corresponds to $g_{RAT} = 1.256$.

Here is an example with $\gamma_1 = -1.0$, $\gamma_2 = -0.5$, $R_T = 40$ km , $R_{REF} = 1$ km, for two values of g_{RAT}: 1.189 and 1.035. The graph below plots gsprd from the SMSIM program *fmrsk_loop_fas_drvr*, along with a direct evaluation of the function for $g_{RAT} = 1.035$ (corresponding to $\xi = 10$). For comparison, the two-segment standard gsprd function is also shown. The $g_{RAT} = 1.035$ results are almost the same as the standard function. As a check of the program *fmrsk_loop_fas_drvr,* the spreading is shown for two values of oscillator period. As the figure shows, the spreading is independent of period (as it should be).



e^(-1.0*ln(x/1)+(-0.5+1.0)*(1/10)*ln(((x/1)^10+(40/1)^10)/(1+(40/1)^10)))
standard gsprd
grat = 1.035, T = 0.5
grat = 1.035, T = 5.0
grat = 1.189, T = 0.5
grat = 1.189, T = 5.0
1
0.2
0.1
0.02
0.01
1
2
10
20
100
200
gsprd
Rps
e: C:\smsim\gspread_generalize_stafford\fmrsk_loop_fas_drvr.plot_gsprd.draw; Date: 2021-02-25; Time: 16:54:09

Figure 01.

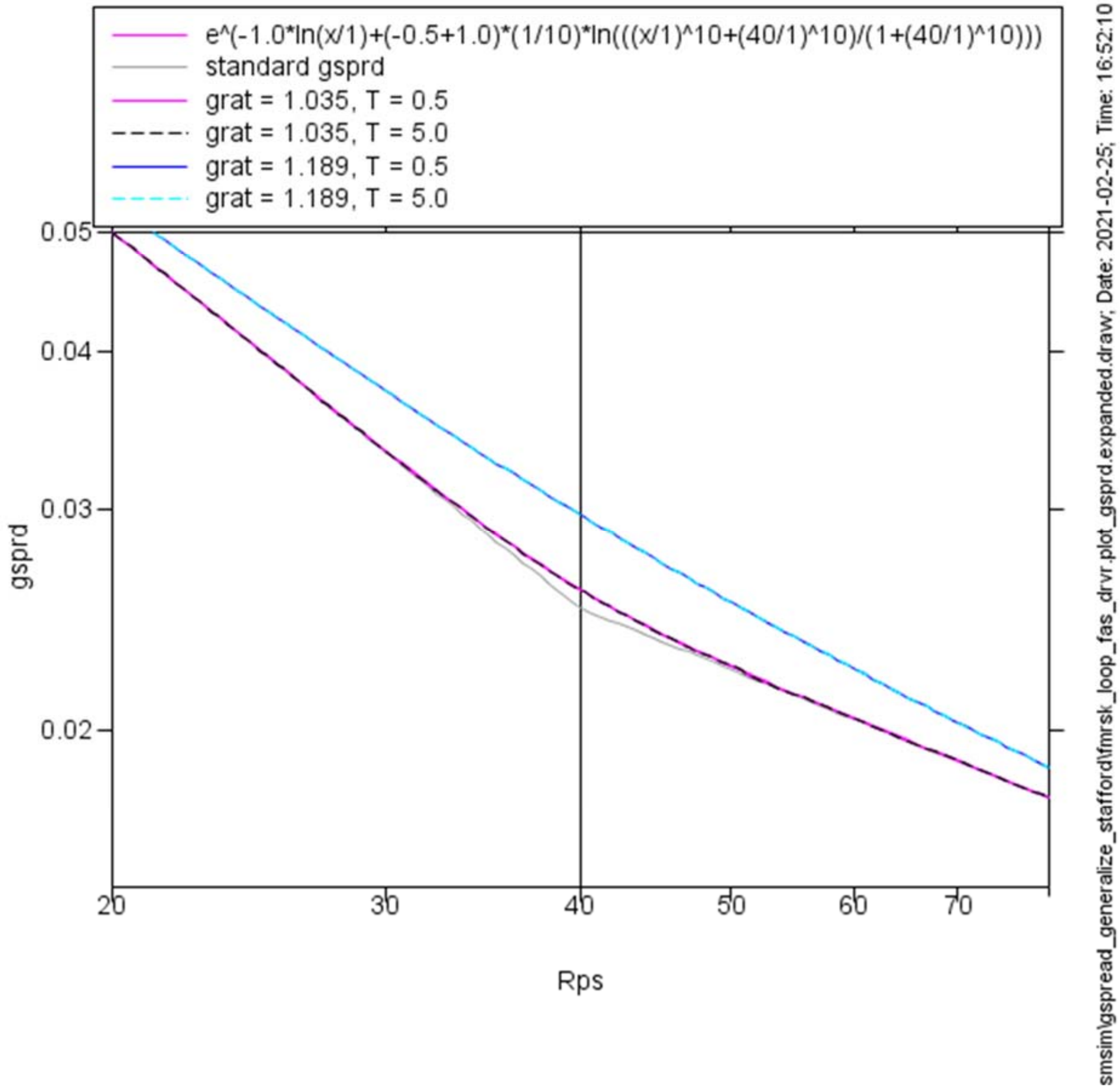


Figure 02. This is an expanded version of Figure 01, showing more detail near the transition.

The two graphs show that the function is working properly.