# Representing the Boore and Thompson (2015) finite-fault factor by a single equation

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Boore and Thompson (2015) (BT15 hereafter) gave the following equation for the finite-fault factor ($h$, but sometimes written as f_ff in my SMSIM programs).

$$\log h(\mathbf{M}) = \begin{cases} a_1 + b_1(\mathbf{M} - M_{t1}) & \mathbf{M} \le M_{t1} \\ c_0 + c_1(\mathbf{M} - M_{t1}) + c_2(\mathbf{M} - M_{t1})^2 + c_3(\mathbf{M} - M_{t1})^3 & M_{t1} < \mathbf{M} < M_{t2} \\ a_2 + b_2(\mathbf{M} - M_{t2}) & \mathbf{M} \ge M_{t2} \end{cases} \quad (1)$$

This is equation (3) in BT15, with the correction that $h(\mathbf{M})$ on the left side of BT15 equation (3) should have been $\log h(\mathbf{M})$. The coefficients are given in Table 1 of BT15. After the publication of BT15, I realized that only five coefficients are needed to specify the model: these are the equations for the two line segments and the range of **M** over which the transition takes place. A subroutine in the SMSIM file *rv_td_subs.for* reads these five coefficients and then computes the coefficients needed for the BT15 equation (3). In the SMSIM program params file, the input coefficients are those in these equations:

Line 1:

$$\log h = c_1 + c_2 \mathbf{M} \quad (2)$$

Line 2:

$$\log h = c_3 + c_4 \mathbf{M} \quad (3)$$

and width of the transition $\Delta\mathbf{M}$. For some reason that I no longer remember, I used the same coefficient names for different things in BT15 and in SMSIM. I will use those in equations (2) and (3) here.

In an as-yet unpublished report, Peter Stafford uses a single equation to provide a smooth transition between two geometrical spreading lines (the equation gives the log of the geometrical spreading function, the function being R to a power for each line). I generalized his equation to allow a range of transition distances. My notes discussing this are in the file *C:\smsim\gspread_generalize_stafford/generalize_staffords_gspread.v2.docx*. I realized that the same single equation can be used to approximate the BT15 tripartite equation. Five coefficients are still needed: the equations of the two lines (four coefficients) and a parameter $h_{RAT}$ which is a measure of the ratio of the smooth $h(\mathbf{M})$ to the $h(\mathbf{M})$ of the lines at the magnitude where the two lines intersect ($\mathbf{M}_T$). $h_{RAT}$ should be less than unity if line 2 has a flatter slope than line 1, as it does in BT15. The closer $h_{RAT}$ is to unity, the sharper the transition. After presenting the equations, I show results for $h_{RAT}$ leading to a close approximation to the BT15 results for the finite-fault factor. The equation for $h(\mathbf{M})$ is given by:

$$\log h(\mathbf{M}) = \log h_T + c_2\left(\mathbf{M} - \mathbf{M}_T\right) + \left(c_4 - c_2\right)\frac{1}{\xi}\log\left[\frac{10^{\xi(\mathbf{M}-\mathbf{M}_T)}+1}{2}\right] \quad (4)$$

where

$$\mathbf{M}_T = \frac{(c_1 - c_3)}{(c_4 - c_2)} \quad (5)$$

and

$$h_X = 10^{(c_1 + c_2 \mathbf{M}_T)} \quad (6)$$

$$h_T = h_{RAT} h_X \,, \quad (7)$$

and

$$\xi = \frac{(c_4 - c_2)\log 2}{\log h_{RAT}} \;. \quad (8)$$

Note that the equations can be written in terms of natural logs by substituting "ln" for "log" and "e" for "10", and the coefficients for the two lines must be changed accordingly (the base 10 coefficients must be multiplied by ln(10)=2.303 if equations (2) and (3) are in terms of natural logs (base e)).

The above gives general equations, but for equivalence to BT15, I use the BT15 coefficients for both lines, as given in Table 1.

Table 1. The coefficients of the line segments used in BT15.

| log base | c1 | c2 | c3 | c4 |
|---|---|---|---|---|
| 10 | -1.720 | 0.430 | -0.405 | 0.235 |
| e | -3.960 | 0.990 | -0.933 | 0.541 |

and then choose $h_{RAT}$ to achieve a close approximation to BT15. With coefficients in Table 1, the transition magnitude is

$$\mathbf{M}_T = 6.74359 \quad (9)$$

I initially chose $h_{RAT} = 0.8939$ based on the ratio of the BT15 smooth *h* at the transition magnitude to the *h* value of the individual lines at the transition (cross-over) magnitude. This gave the following revision of Figure 1 in BT15:

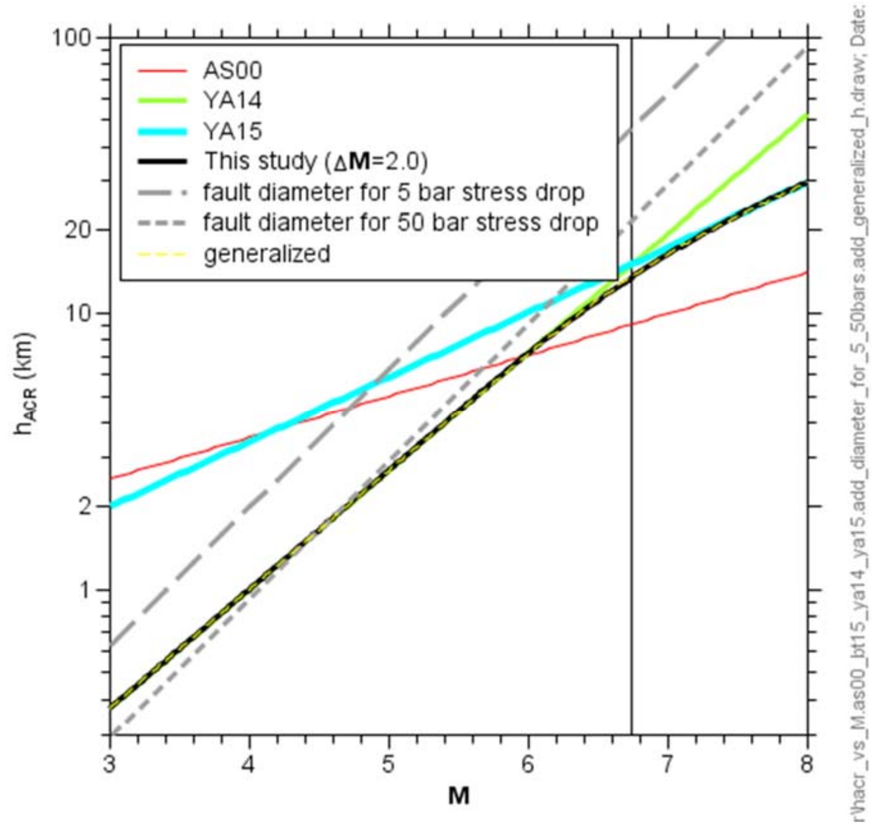


Figure 1.

The single equation for *h* given by equation (4) is shown by the dashed yellow line. It is virtually indistinguishable from the BT15 line (solid black). To see the comparison in more detail, Figure 2 shows the ratio of the new *h* and the BT15 *h*, as a function of magnitude:

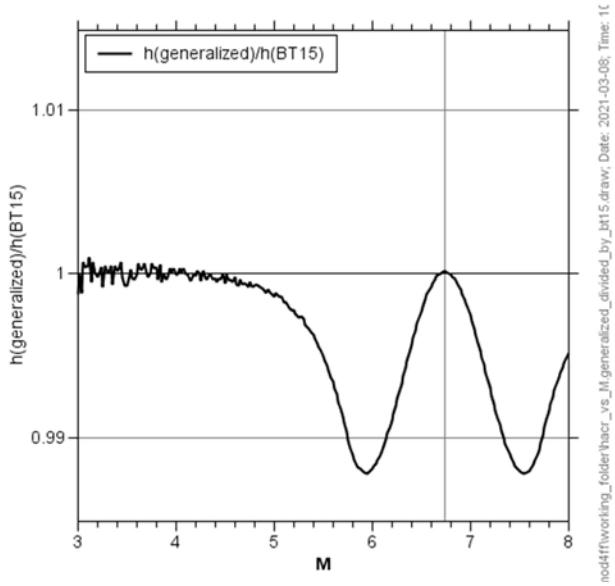


Figure 2. Using $h_{RAT} = 0.8939$

By design the ratio is unity at the transition magnitude (that magnitude is indicated by the vertical black line). By using different values of $h_{RAT}$ I found a value that balanced the two troughs in the ratio on either side of the transition magnitude and the value of ratio at the transition magnitude. This revised value of $h_{RAT}$ was $h_{RAT} = 0.9015$, giving the following ratio as a function of magnitude:

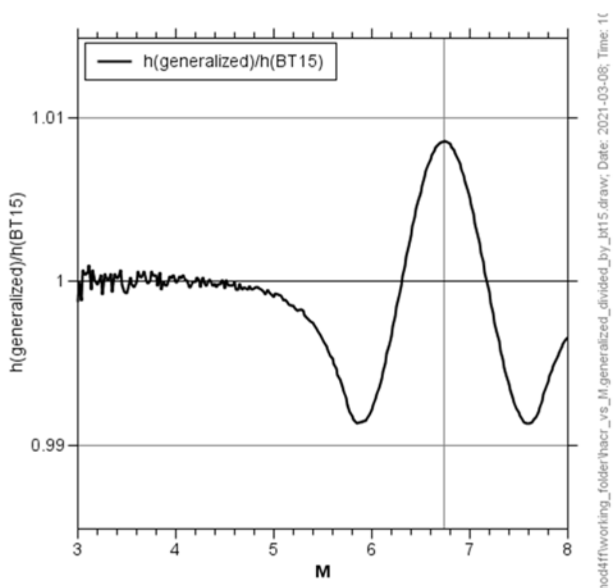


Figure 3. Using $h_{RAT} = 0.9015$

In practice either value of $h_{RAT}$ would give results using the single equation given in these notes equivalent to those using the tripartite BT15 equations.

In summary, here are the coefficients for the single equation that is equivalent to the BT15 equations for active crustal regions (ACR):

Table 2. Coefficients of single equation giving finite-fault factors equivalent to the BT15 tripartite equation for active crustal regions.

| log base | c1 | c2 | c3 | c4 | hrat |
|---|---|---|---|---|---|
| 10 | -1.720 | 0.430 | -0.405 | 0.235 | 0.9015 |
| e | -3.960 | 0.990 | -0.933 | 0.541 | 0.9015 |

Reference

Boore, D. M. and E. M. Thompson (2015). Revisions to some parameters used in stochastic-method simulations of ground motion, *Bull. Seismol. Soc. Am.* **105**, 1029–1041.